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Autonomy, Inquiry and Mathematics Reform

Andy Carter
Roosevelt University

A growing body of research provides evidence that the National Council of Teachers of Mathematics *Standards* (1989, 2000) can have a positive impact on student learning (Schonfeld, 2003; Senk & Thompson, 2003; and Hiebert, 2003). As Hiebert notes "where these programs have been implemented with fidelity for a reasonable length of time, students have learned more and learned more deeply than in traditional programs" (p. 20). Examples of successful *Standards*-based programs have been characterized by some as "inquiry mathematics" (Cobb & Bauersfeld, 1995; Richards, 1996; Yackel, 2003). The theoretical basis for the *Standards* is closely associated with Piaget's theory of constructivism (Steffe & Kieren, 1994; Yackel, 1996,). Piaget's (1948/1973) goal for education, the development of autonomy, is closely associated with inquiry. In this paper I will explore the connections between these two ideas, the development of autonomy and inquiry mathematics.

Inquiry Mathematics

What are the characteristics of *Standards*-based instruction in mathematics? Richards (1996) and Cobb and Bauersfeld (1995) address this question by making a distinction between "school mathematics" and "inquiry mathematics." School mathematics is what occurs in traditional mathematics classrooms. What is learned in school mathematics is useful for solving familiar and routine problems. Classroom activities generally involve computation or symbolic manipulation directed toward finding a specified result which has been pre-determined by the teacher. A transmittal model of learning is employed and there is little or no negotiation of mathematical meaning. Learners are not viewed as active agents in their own learning, but are instead viewed as passive recipients of a well-defined, objective body of knowledge.

In an inquiry mathematics classroom, students are actively engaged in the construction of knowledge. A classroom micro-culture has been established where students and teachers work cooperatively to constitute a community of inquiry and validation. Classroom activities are characterized by listening, discussions, mathematical justifications, and “dialogical encounters,” in which “one begins with the assumption that the other has something to say to us and to contribute to our understanding” (Bernstein, 1992, p. 337). From an observer’s perspective, teachers and students appear to be acting in a mathematical reality based on the mutual construction of social norms and shared meanings. This reality is constantly being elaborated and changed through the negotiation of meaning. Students are viewed as active agents in a situated, social, transactional learning process. But what are the connections between inquiry mathematics and autonomy? These connections will become more apparent in the following discussion of the role of autonomy in education.

Autonomy and the Aims of Education

According to Jean Piaget (1948/1973), the central purpose of education is the development of autonomy. Kamii (1994) describes Piagetian autonomy as “the *ability* to think for oneself and to decide between right and wrong in the moral realm and between truth and untruth in the intellectual realm by taking all relevant factors into account, independently of rewards and punishments” (p. 673). DeVries (1987) provides a more succinct interpretation of Piagetian autonomy as “the capacity to create rules” (p. 32). In other words, autonomous individuals have gone beyond the ability to simply follow rules and have developed the capacity for creating rules for themselves to deal with specific situations. Both of these definitions reflect Piaget’s belief that individuals can develop the capacity to make well-informed intellectual and moral decisions in the absence of external controls, such as punishment and rewards. In fact, Piaget believes that such external controls are an impediment to both moral and cognitive development. More recently, authors such as Deci (2001), Glasser (1998), and Kohn (1993) provide support for Piaget’s position by presenting powerful examples of the negative impacts of punishment and rewards on human development.

For Piaget, socio-moral and cognitive development constitute two inseparable aspects of human development (DeVries, 1997). According to Piaget, children are assisted in their transition from heteronomy (regulation by others) to autonomy when adults limit coercion and control and replace these interventions with interactions of mutual respect and cooperation. Cooperation here refers to a type of social interaction where individuals are "striving to attain a common goal while coordinating one's own feelings and perspectives with a consciousness of another's feelings and perspectives" (p. 5). For optimal development, children need a social context, which provides cooperative relationships with other children and adults. Piaget also theorizes that cooperative relations with adults, which are characterized by mutual respect where adults seek to minimize their use of authority, are of particular importance because they provide opportunities for children to practice governing their own behavior (DeVries, 1997).

The Piagetian view of the development of autonomy as the aim of education is consistent with Dewey's progressive views: "The ideal aim of education is creation of power of self-control" (1938, p. 64). Dewey's use of "self-control" is analogous to the Piagetian notion of autonomy. This connection is made more explicit in Dewey's reference to the potentially harmful effects of external controls: "The kind of external imposition which was so common in the traditional school limited rather than promoted the intellectual and moral development of the young" (p. 22). Finally, along with Piaget, Dewey emphasizes the central role played by cooperative work in human development. Dewey envisions the classroom as a community where students and teachers engage in cooperative activities, where order is established by "the moving spirit of the whole group . . . The teacher reduces to a minimum the occasion in which he or she has to exercise authority in a personal way . . . The plan in other words, is a co-operative enterprise, not a dictation." (p. 54). Both Dewey and Piaget recognize and emphasize the importance of cooperative activities in human development.

The development of autonomy also plays an important role in the sociocultural theory of learning associated with Vygotsky (Forman, 2003). While much has been made of Vygotsky's criticisms of Piaget, many of these critiscms can be attributed to either a misunderstanding of Piaget's theories or Vygotsky's historical context: "he [Vygotsky] still made insightful pedagogical arguments with which Piaget would have agreed in principle" (Hsueh, 2002 p. 12). In "A Sociocultural Approach to Mathematics Reform: Speaking, Inscribing, and Doing Mathematics Within Communities of Practice," Forman argues that to understand Vygotsky's theory of development, one needs to carefully study the social factors that impact "how a novice's performance becomes less dependent on *other-regulation* (regulation by others) and becomes more self regulated over time" (p. 334). In other words, to understand Vygotsky's theory of human development, one needs to look closely at the ways in which individuals develop autonomy.

Deci (2001, 1995), another psychologist who has written extensively about the importance of autonomy, describes autonomy as acting volitionally with a sense of choice and a willingness to behave responsibly in accordance with one's interests and values (1995, p. 9). A key aspect of Deci's definition is the importance of choice: "Providing choice, in the broad sense of the term, is a central feature in supporting a person's autonomy" (1995, p. 34). Deci, like Piaget, recognizes the fundamental drive of children to make sense of their world. Piaget characterizes this drive as the "fuel for the constructive process" (DeVries, 1997, p. 14), and Deci comments that "a child's curiosity is an astonishing source of energy" (1995, p. 18). Both men see the strong connections between developing autonomy and using a child's natural curiosity as "fuel" for intellectual and moral growth.

Reform curricula in mathematics have sought to tap into this reservoir of natural curiosity by utilizing inquiry-based approaches to instruction. But for inquiry mathematics to succeed, teachers need to recognize the connections between inquiry and autonomy. Inquiry can be encouraged, stimulated, and aroused, but it cannot be forced because it is a volitional activity. For inquiry to occur, students must first have the opportunity to choose to engage in it. Then they must also have the capacity to take the relevant factors into account in making the decisions necessary for enacting the inquiry. The conditions, which support the development of autonomy, opportunities to make choices, and to work cooperatively with others, also support inquiry. The two, inquiry and autonomy are inextricably linked. Or, to build on Piaget's metaphor, inquiry is the fuel that fires the engine of human development.

Although the ideas of inquiry mathematics have been widely accepted since the publication of the *Standards* (1989), fundamental changes in instructional practices have been slow in coming about. As Hiebert (2003) observes in a recent meta-analysis of current classroom practices, “Today, teachers continue to teach [mathematics] much like their forbears did” (p. 11). One approach that has been suggested for addressing this apparent resistance to change is to find and carefully document classrooms that have successfully adopted inquiry-based instructional practices (National Research Council, 2001; Hiebert, 2002). In the next section I present a descriptive narrative which focuses on the social interactions occurring during an inquiry-based mathematics lesson.

Case Study: Data and Methodology

The lesson presented here is taken from a one-year case study of an urban small-school as it implemented an inquiry-based mathematics curriculum, *Math Trailblazers* (1997). During this time, I made regular trips to the school to assist teachers in the curriculum’s implementation. I also attended weekly faculty meetings where I was able to develop personal relationships with the teachers and a better understanding of them and of the goals they had for their school.

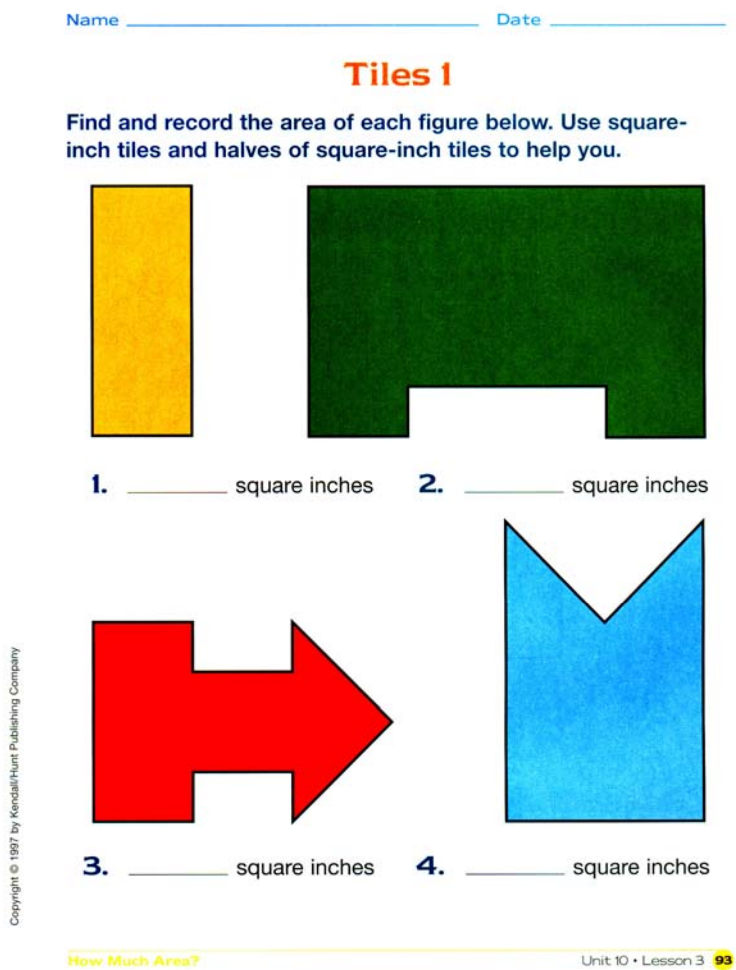
For the case study, formal teacher interviews from the beginning, middle, and end of the school year were recorded and transcribed. Informal interviews were also conducted in conjunction with classroom observations and weekly faculty meetings. During March and April, teachers were videotaped teaching at least one complete *Math Trailblazers* lesson. These tapes were then transcribed and used in developing detailed descriptive narratives of the lesson as a first iteration of data analysis (Anfara, Brown, & Mangione, 2002). “Dialogical data gathering” (Carspecken & Apple, 1992, p. 531), where teachers participated in data analysis, was then accomplished by providing each teacher with the narrative and the videotape from the lesson. The narratives were discussed at length and revised to incorporate teacher input to form a second iteration of data analysis. Teacher perceptions and input provided an important source of triangulation adding to the reliability of the case (Stake, 1995).

Lesson Narrative

The following narrative comes from a first grade math lesson taught by Maria. While teaching either kindergarten or first grade over the past seven years, Maria has developed a teaching style that is both caring and serious. Students have learned that she is a sympathetic listener and students frequently come to her with their concerns. At the same time, she also has established high expectations for student participation in classwork and clear boundaries of acceptable student behavior. Although the interactions seen in this lesson are typical for Maria's class, it should be noted that they reflect a seven-month evolution of social norms in a classroom where listening and cooperation are valued, modeled, and practiced on a daily basis. Evidence of this evolution appears in notes from one of my first observations in Maria's classroom: "Many students don't seem to know what to do…things seem a little chaotic." At that point I was concerned about Maria's competency as a teacher and I wondered why she wasn't being more explicit in telling her students what to do. Later, I realized that what I had interpreted as a lack of direction was more precisely an early opportunity for student decision-making.

The episode described here comes from a first grade *Math Trailblazers* (Wagreich, 1997) lesson involving the measurement of area. It is important to note that the primary intent of the lesson is not to teach the concept of area, but instead to use the idea of area in establishing a problem-solving context. In the lesson, students are asked to find the area of various shapes by covering them with tiles (see Figure 1). This simple problem-solving situation—students covering flat shapes with tiles—provides an opportunity for students to actively engage in *doing* mathematics and in communicating their results to others.

Maria begins the lesson with a brief discussion of the previous day's work, then uses a student's comment about "one half of a tile" to lead into today's lesson. She informs the class that they will be finding the area of more shapes and that they will be using both square-inch and half-square-inch tiles to cover the different shapes. She then demonstrates on the overhead projector how the rectangular halves (RHs) and triangular halves (THs) can be put together to make a whole tile. Maria then hands out the sets of plastic square-inch tiles and an envelope containing the two kinds of half-square-inch pieces (see Figure 1), and has students begin work on page 93. Over the next 10 minutes students work on problems 1 and 3 and volunteers demonstrate their answers on the overhead. Once these have been discussed, Maria asks the class to complete the problems on pages 93 and 94 (a page with problems similar to those found in figure 1).



Name ____________ Date ____________

Tiles 1

Find and record the area of each figure below. Use square-inch tiles and halves of square-inch tiles to help you.

1. ______ square inches **2.** ______ square inches

3. ______ square inches **4.** ______ square inches

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How Much Area? Unit 10 • Lesson 3 93

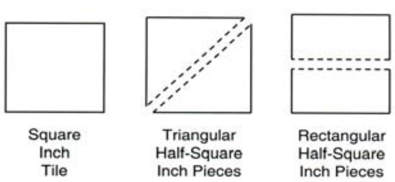

Figure 1

From Math Trailblazers 1st Edition, a TIMS Curriculum from the University of Illinois at Chicago. Copyright c 1997 by Kendall/Hunt Publishing Company. Reprinted with permission.

For the next twenty-three minutes the class bustles with activity as the students work on the assigned problems. During this time, Maria does not speak to the class as a whole but instead moves around the room assisting students. Most interactions are student-initiated, one-on-one conversations where Maria listens carefully and responds directly to students’ ideas and questions. Often, other students who are nearby are also listening, watching, and sometimes participating in the exchange. The following vignette illustrates a set of typical interactions during the lesson.

Julia walks up to Maria and tells her the answer she has found for problem 4. Maria responds, “Five and a half square inches? Wow! I am going to come right over Julia, so you can show me the five and a half square inches.” Maria spends a little over four minutes working with Julia and her group. Most of the time is spent helping Julia distinguish between whole and half tiles. When Maria leaves, it is not clear how well Julia understands the difference between the two. It is clear, however, that Julia had Maria’s attention as they worked together trying to make sense of each other’s thinking. This type of dialogical interaction occurred frequently throughout the lesson.

While Maria has been working with Julia and her group, the rest of the class has been busy. Enrique and Armando, who are standing side-by-side, present an interesting contrast. While Armando works in a serious and determined manner, Enrique is much more animated and he hums a tune as he works. Soon, he begins a little dance. It came about quite naturally, as he was reaching over for the envelope on Armando’s desk. First his shoulders began moving back and forth as he was reaching out, and this was followed by a shuffling of his feet. Then holding his hands up high and softly snapping his fingers he appears to be doing a Salsa version of the Twist. Enrique’s brief dance ends as abruptly as it starts as he seamlessly goes back to his work. A moment later Enrique looks up at Armando with a sly grin. Armando glares back at him, shakes his finger disapprovingly and then they both return to their work.

Ricardo and Luis are working together at the same table across from Armando and Enrique. Luis is still working on problem 3 and he appears to be having trouble. He has the three square tiles in place and he takes one of the THs from Ricardo’s desk. He tries unsuccessfully to fit the piece into the figure. Looking perplexed, Luis asks Ricardo a question, picks up an RH and starts trying to fit it onto number 3. Ricardo, a physically strong and somewhat temperamental child, watches Luis work until, losing patience, he snatches the RH from Luis's hand and returns it to the envelope. He then finds a TH and carefully positions it on number 3. Luis searches through the envelope to find another TH, which he uses to finish covering the figure.

The assistance that Ricardo provides for Luis is not done in a particularly cooperative spirit, but his intent does appear to be to help Luis find a solution to number 3, and perhaps it is the best that Ricardo can do. At the very least, his focus on the work indicates his involvement in the assigned task and his acceptance of some responsibility for helping his partner.

The class has now been working on pages 93 and 94 for about 10 minutes and there is considerable noise and movement in the room. Almost one-third of the students are out of their seats. Some are at the overhead working to cover the figures on the transparency of page 93. Others are standing near Maria listening or waiting for a chance to ask her a question. Armando waits only a short time before Maria turns her attention to him. He reports to her that he has finished pages 93 and 94 and asks what he should do next. After a few moments of thought, Maria responds.

> Maria: You could do page 95. But you know what? What you might want to do is walk around and compare your answers to other people's answers.
>
> Armando: Do I tell them the answer?
>
> Maria: No. Don't tell them the answers, but you can make sure that they are the same. Everybody has the same shapes, so just compare to see if they have the same answers and if not, maybe one of you guys made a mistake. Right?

With book in hand, Armando then moves around the room comparing and discussing his answers with other students.

Nearby, the overhead continues to be a center of activity and fascination. The tiles of the last group who worked there have been pushed aside, and two girls begin working on number 3. They quickly put the three wooden tiles into place and then begin twisting and turning two THs, trying unsuccessfully to fit them into the figure. They appear to be convinced that they need to use the THs, because they continue working with them rather than trying any RHs. The two girls are so engrossed in their work that they are oblivious to what is going on around them. Two boys standing behind them are casting shadow shapes on the overhead screen with their hands. Another boy is experimenting with the overhead. He places a sheet of paper below the light creating a blank screen where there were once tiles and talking ducks. The boy removes the paper, revealing that the girls are still trying to fit the two THs into number 3. When he again blanks out the screen, Mark, who is standing nearby, pushes him aside with apparent distaste for his unruly behavior. Mark then begins working with the two girls and together they align the THs to cover number 3.

Luis returns to his seat by Ricardo and is closely followed by Armando. Armando stands behind where they are sitting and inspects their work. Soon Armando and Luis are having a heated discussion. Apparently Armando has made a suggestion that angers Luis, and he gets up and gives Armando a push. Undeterred, Armando leans over, and pointing at something in Ricardo's book, makes another comment. Ricardo immediately jumps out of his seat and begins arguing with Armando. Ricardo emphatically points at the overhead in his defense of his answer. Armando looks at the overhead, then back at his book, and shakes his head. He stands with an uncharacteristically bewildered look. Ricardo walks over, hits him solidly on the shoulder and then, with a triumphant swagger, Ricardo returns to his seat. Eventually, Armando is able to confirm with Maria that the answer on the overhead is incorrect and goes up to correct it.

Discussion and Conclusion

The twenty-three minutes of uninterrupted work time from which this vignette was taken is important for several reasons. First, it allowed enough time for Maria and her students to engage in dialogical interactions where she and her students both appear to assume that the other has something worthwhile to add to the conversation. These interactions exemplify the type of listening and negotiation of meaning that occur in inquiry mathematics. Additionally, these conversations provide the teacher with valuable informal assessment data while also allowing "the pupil, in effect, to become a party to the negotiatory process by which facts are created and interpreted" (Bruner, 1986, p. 127). By engaging in conversations with her students, Maria also models listening as a powerful and often underutilized tool for the construction of mathematical meaning (Davis, 1996).

Finally the long duration of this uninterrupted period allows enough time for students to get off task and then self-initiate their return. Examples of this were the constantly changing mix of play, experimentation, and investigation occurring at the overhead, or when Luis did his little dance and then chose to return to his work. Having the opportunity to make such choices is important in the development of student autonomy, which is critically important to the learning process (DeVries, 1997; Kamii, 1994). In a review of recent research, Rodgers (1998) observed, “Research indicates that more autonomous children have greater classroom competence and are less likely to act out in class. They also have higher achievement scores and grades” (p. 78). During the lesson, many of Maria’s students were off task at one time or another. But most, if not all, of these students also chose to return to their assigned work. Providing students an opportunity to make such decisions is also giving them the opportunity to grow both intellectually and emotionally (Deci, 2001; Glasser, 1998; Kohn, 1993).

The active intellectual and emotional engagement of Maria’s students provides a powerful example of the potential benefits of inquiry-based instruction in mathematics. But the wide disparity between her teaching and what is happening in most U.S. classrooms also raises important questions. One concerns the feasibility of the widespread adoption of inquiry-based instruction. Or as Kamii (1998) asks, “Why do educators keep doing the traditional [reward and punishment] things that are not working” (p. 6)? This is essentially a question about what we value. Bringing about the changes necessary for the wide scale implementation of inquiry-based instruction will require a massive commitment of time and resources (Hiebert, 1999). That in turn will require that we broaden what we value to include developing the capacity for self-regulation or autonomy.

Another important question concerns Maria’s role in creating a classroom environment supportive of inquiry. Maria shed some light on this question in the following comment she made during a faculty meeting: “I just wanted to add this. It is our role to prepare the environment for the children to be able to have that control over their learning. And it is a constant thing that you have to keep rearranging and you have to keep checking to see if this works or that works or if it doesn’t work.” Maria’s point here is about providing students with opportunities to make meaningful choices, to self-regulate, to become more autonomous. She observes that it requires constant effort and close attention to detail to maintain an environment that has achieved a workable balance between order and the freedom to make choices.

My observations and experiences in Maria’s classroom have had a deep impact on my understanding of teaching and learning. The lesson that stands out most vividly for me is the many challenges faced by teachers who seek to create an educational environment that supports inquiry and autonomy. The task is difficult because: it is complex and situational; it requires that teachers listen to and learn from their students; it requires that teachers have enough confidence and self-esteem to share authority with their students; it rejects the commonly held assumptions about the necessity of rewards and punishment; and it is easier to cover content than to negotiate meaning. It is difficult because it requires a questioning of our most deeply held assumptions about teaching, learning, and schools. But in light of these challenges Maria’s work and the work of the many other teachers who have adopted constructivist practices is encouraging. They not only add to existing evidence supporting inquiry-based instruction, they also provide further insights into the complex connections between inquiry and autonomy.

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