Annual Conference
Jack London Square
Oakland, California
October 16–17, 1998
Don’t miss
the 1998 ACT Conference!
October 16–17, 1998

Waterfront Plaza Hotel, Jack London Square,
Oakland, California
(See page 22 of this magazine for registration information.)
THE CONSTRUCTIVIST
Volume 12, Number 3 Fall 1997

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FALL 1997
Dear Members:
I hope all of you are off to a great start for the new academic year and are making plans to attend our Annual ACT Conference in October. The program looks terrific, with powerful keynote speeches from Geoffrey Saxe and Marilyn Watson. The break-out sessions will address a range of ages, issues, and practices in constructivist teaching. Presenters for these workshops come from eleven different states crossing the US. I am sure you will agree that this impressive group offers a variety of content that will appeal to a broad range of interests.

Paul Ammon, Past President of ACT, and Jill Lester, Program Chair, have been hard at work to organize an outstanding program and meeting arrangements that will delight everyone. The Waterfront Plaza Hotel offers fresh air waterfront meeting rooms, guest rooms with spectacular harbor views, and a restaurant overlooking the bay. It is located on the bay at Jack London Square, where you will find more than a dozen great restaurants and 46 shops to visit.

You might notice that several meals and a reception are included in the registration fee. I personally look forward to these events as great opportunities to meet and network with educators from across the country. I hope you will be there and be ready to enter several new names in your address book.

The articles in this issue of The Constructivist once again offer us excellent provocations to reflect on teaching practices that affect the construction of knowledge among children and adults. Kamii, Pritchett, and Nelson present a detailed analysis of a 45-minute math discussion in a fourth grade classroom. It beautifully illustrates constructivist teaching practices that encourage children to take initiative in solving mathematical problems. The detailed documentation of this episode enables us, the readers, to participate with the authors in studying the thinking of these young children. The article also describes the particular strategies used by the teacher to help these children make their ideas visible. The authors have succeeded not only in building their case about the depth and richness of children’s thinking that comes from such kinds of learning experiences, but they also model and illustrate the processes of collective reflection that support learning among children and adults.

In the second article, Kathleen Martin’s research illustrates how valuable Piagetian conservation tasks can be in helping prospective teachers to recognize the complexity of children’s reasoning and the implications for teaching practices that focus on cultivating children’s natural powers of organization. Teachers and teacher educators alike will find this article helpful in reflecting on learning contexts for children and adults.

I look forward to seeing many of you in Oakland. At the brief business meeting that concludes the conference, the board members and I will be soliciting your ideas and interests about future directions for ACT. If you cannot attend, please send your thoughts to me at Webster University, 470 E. Lockwood, St. Louis, MO 63119-3194 or by e-mail <fyfeb@websteruniv.edu>.

—Brenda Fyfe
Dear Readers:

Well, summer has finally come to an end, and I imagine that most of you are probably beginning, or preparing to begin, a new school year. Perhaps some of you participated in some exciting courses this summer, or perhaps you travelled, or read some exciting books. For the first time in a long, long time, I really took a vacation myself, spending five weeks this summer on the coast of Maine. Although I sailed, hiked, and painted, I also had time to ponder and reflect on the work we are all about, the work the authors write about in this magazine. I was reminded of how powerful and renewing the process of reflection can be, how important I believe the work is, and how crucial it is to network and share with each other.

Although we can’t always get so much time off as long summer vacations provide, writing can be a powerful way to reflect, and, of course, a way to share and network. This issue provides such reflections on both theory and practice.

I want to encourage others of you to take time and reflect ... to write about your practice, your children’s work, or the connections you are making between theory and practice. Many projects based on constructivism currently exist across the country. One only needs to take a look at the ACT Conference program to see the varied workshops and the representation from across the United States. Tell us about your projects, your struggles, your successes. How has constructivism informed your practice?

As you can see from the articles previously published, we seek manuscripts written in an informal style, approximately 10 typed pages, preferably with photographs. I would like to put together special issues on writing process and emergent literacy, science, mathematics, project work and inquiry, school change, and assessment. Writing on other topics is encouraged as well, as long as the general theme is the relation of constructivism to classroom practice.

Manuscripts should be sent to me (original and two copies) at the following address: Association for Constructivist Teaching, NAC 3/217, City College of New York, 138th Street and Convent Avenue, New York, NY 10031.

—Catherine Twomey Fosnot
Classified Ads

Job Market

Assistant Professor, Educational Technology, Webster University.
Status track. Doctorate in instructional technology, educational technology, or related field. Teaching experience using technology in elementary or secondary schools; teaching experience in graduate and/or undergraduate levels; expertise in multimedia, distance learning, courseware authoring, or programming desirable.

Send application, resume, transcripts, and three references to: Dr. Andrea Rothbart, Search Committee, School of Education, Webster University, 470 E. Lockwood, St. Louis, MO 63119-3194. Applications will continue to be accepted until the position is filled.
Address e-mail inquiries to: rothbart@websteruniv.edu

Letters to the Editors

The editors of The Constructivist want your feedback! Please send all Letters to the Editors to Catherine Twomey Fosnot, The City College of New York, NAC 3/209a, 138th Street and Convent Avenue, New York, New York 10031.
52 x 8: The Importance of Children’s Initiative

Constance Kamii, Michele Pritchett, and Kristi Nelson

Problems such as 52 x 8 are easy for most fourth graders. Almost all of them know the conventional algorithm for getting the correct answer, and fourth graders seldom have trouble with such problems.

However, we have been developing a very different way of teaching mathematics based on Piaget’s theory of constructivism. Piaget showed that children acquire logico-mathematical knowledge not by internalizing rules from the outside but by constructing (making or creating) relationships from within, in interaction with the environment. We, therefore, give problems to children and ask them to do their own thinking to solve them in their own ways (Kamii, 1989a, 1989b, 1990a, 1990b, 1994).

Children’s inventions to solve multidigit multiplication problems have been described elsewhere (Kamii, 1990a, 1990b, 1994), and the focus of this article is on another aspect: the depth and richness of children’s numerical thinking that comes out of their initiative. We describe what happened in a 45-minute session in a fourth-grade classroom and conclude with ways in which we foster the development of initiative.

The second author was the teacher who led the 45-minute discussion. She began the math hour one day in late October by reminding the class that the problem of the previous day was 38 x 7.

As she wrote the following problem on the board, she said, “Today, I want you to solve this problem in your head, without writing anything (pause), except the answer if you think you might forget it.”

52 x 8

When most of the hands were up, the teacher listed on the board as usual all the answers the children volunteered: 416, 308, 564, 418, and 466. She first asked, as always, if anyone had any comment about answers that seemed reasonable or unreasonable. One child remarked that 308 was not possible because 52 x 6 = 312. Another student argued that the answer had to end with a 6, since 2 x 8 = 16. Everybody agreed with these comments, and the teacher crossed out all the answers except 416 and 466.

Five students then explained the five procedures shown in Figures 1–5. The first four methods were typical of what the class had been doing, and the class agreed on the answer of 416.
I know that 4 quarters make $1.00. So 8 quarters is $2.00. Double that because you need 2 quarters to make 50 cents, and that’s $4.00. Plus 16 cents because $2 \times 2$ cents = 16. So the answer is $4.16, 416.$

The idea of quarters (Figure 5) inspired one student, Cathy, to think of an unusual procedure. “I can change the problem to $26 \times 16$,” she announced. Several students asked for an explanation, and Cathy confidently said, “If you take only half of 52, you have to times it (26) twice as many times . . .” She also argued as follows that the answer to $26 \times 16$ was the same as the one the class had been getting:

\[
\begin{align*}
26 \times 10 &= 260 \\
26 \times 6 &= 26 \times 3 &= 78^* \\
&\quad + 78 \text{ (to double 26 \times 3)} \\
&\quad 156 \\
&\quad 260 \\
&\quad 416
\end{align*}
\]

* The reader may object to use of the “≈” sign here. This sign was used incorrectly from the standpoint of adult conventions, but not from the point of view of children’s representation of their ideas. For example, when children hear that some people were created more equal than others, they cannot represent to themselves the same meaning as adults can because they cannot put into the words the same ideas as adults. When children become able to think more like adults, they will easily become able to use the “≈” sign, the word “equal,” and every other abstract word more like adults. In other words, representation consists not of the correct use of conventional signs, but of each person’s projection of his or her own ideas into signs at his or her own level. This is why we do not interrupt children’s thinking to teach the adult meanings of written and spoken signs.
Many students remained unconvinced by Cathy’s argument, and the teacher offered an explanation with smaller numbers: $12 \times 2$. “Cathy is saying that if you change the 12 to half of 12, which is 6, you have to add 6 twice as many times as before. That’s why she said she could change $12 \times 2$ to $6 \times 4$,” the teacher explained as she wrote the following numbers on the board:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$12 \times 2$</td>
<td>24</td>
</tr>
<tr>
<td>$6 \times 4$</td>
<td>24</td>
</tr>
<tr>
<td>$6 + 6 + 6 + 6$</td>
<td>24</td>
</tr>
</tbody>
</table>

Dan was bursting with an idea by this time and said, “We could do half of 26, and that’s 13 x 32.” The teacher was delighted and asked him if he meant the following that could now be seen on the board:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$52 \times 8$</td>
<td>416</td>
</tr>
<tr>
<td>$26 \times 16$</td>
<td>416</td>
</tr>
<tr>
<td>$13 \times 32$</td>
<td>416</td>
</tr>
</tbody>
</table>

The class computed the answer to $13 \times 32$ and agreed that it was 416, too. The teacher then asked, “Can we go up?” and the majority of the class enthusiastically changed $52 \times 8$ to $104 \times 4$, and the latter to $208 \times 2$, then to $416 \times 1$, and then to $832 \times 1/2$. The board now looked as shown in Figure 6.

![Figure 6. Changing $52 \times 8$ by doubling and halving the factors](Image)

Before calculating the answers for the top four lines of Figure 6, the teacher asked the class if there was a pattern in each column of factors.

The calculation became increasingly easy from $104 \times 4$ to $416 \times 1$, but $832 \times 1/2$ was problematic. The more advanced students soon agreed that $832 \times 1/2$ had to mean “$832 \div 2$” because the answer had to be 416.

Kevin then suggested that half of 13 was 6 1/2, and that the next line below would be 6 1/2 x 64. The calculation of this problem was a challenge, but someone announced in no time that $13 + 2 = 6.5$ according to his calculator. “Is 6.5 the same thing as 6 1/2?” the teacher inquired, and someone replied, “That’s like 6 dollars and 50 cents. Six and a half dollars is the same thing as 6.5.” For the benefit of those who were not convinced, the teacher explained why “6.5” means the same thing as “6.50” and “6.500.” In the meantime, a few students were punching 6.5 x 64 on their calculators and announced that the answer was again 416.

Several students then suggested with excitement, “Let’s do 3 1/4 x 128.” When someone asked, “How do you do 3 1/4 x 128?” the answer came quickly by doing $6.5 \div 2$ on the calculator, and then $3.25 \times 128$. The students who were not sure that 6.5 really meant something like $650$ seemed more satisfied when they saw that half of 6.5 was 3.25. They were familiar with “$3.25$” but not with “6.5.” Someone announced that the answer to $3.25 \times 128$ was again 416, and the board looked as shown in Figure 7 when the teacher decided to terminate the activity. A third of the class had been unable to understand the discussion, and the teacher decided to ask later whether or not anybody wanted to work some more to see how far they could go with Figure 7.

![Figure 7. The writing on the board at the end of the discussion](Image)
Children’s Initiative

At the beginning of this 45-minute session, the teacher had no intention of introducing the “Russian Peasant Method,” which is what the class essentially invented. According to this method, 52 x 8 is done by doubling and halving the factors in the following way:

\[
\begin{align*}
52 \times 8 &= 104 \times 4 \\
&= 208 \times 2 \\
&= 416 \times 1 = 416
\end{align*}
\]

The teacher did not have any intention of getting into fractions or decimals either and did not dream of getting into this new territory by jumping into multiplication and division first.

The only initiative the teacher took during the entire session was: (a) in suggesting at the very beginning that the class work on 52 x 8, (b) in asking, “Can we go up?” after two children had “gone down” to 26 x 16 and 13 x 32, and (c) in inquiring if there was a pattern in each column of Figure 6. All the other questions and ideas were initiated by various members of the class.

The 45 minutes described above attests to the desirability of fostering children’s initiative. In the fourth-grade textbook (Hoffer, Johnson, Leinwand, Lodholz, Musser, & Thoburn, 1991), there are two chapters devoted to fractions before a separate chapter on decimals. The addition and subtraction of fractions appear in the fourth-grade textbook, but the multiplication and division of fractions are introduced in fifth grade. Children’s minds do not work in the fragmented manner by which textbook writers organize their texts. Children go much farther and more naturally, with great joy, if they are encouraged to pose their own questions and answer them in their own ways.

The Depth and Richness of Children’s Thinking

The children in the preceding account often related addition to multiplication as can be seen in Figures 1 and 2. In relating 52 x 8 to 26 x 16, they also thought deeply about the sameness of adding 52 eight times and adding 26 sixteen times. Changing 52 x 8 to 26 x 16 also gave them an opportunity to relate division to multiplication in an intriguing way.

Multiplying different numbers that came out of 52 x 8 provided many opportunities to think about multiplication itself. The first problem, 52 x 8, required only the partitioning of 52 into 50 and 2. The next problem, 26 x 16, however, was much more complicated because it involved two two-digit numbers. Some children partitioned only one of the numbers, but others partitioned both numbers as can be seen in Figure 8.

The most impressive numerical thinking was in children’s natural entry into fractions and decimals. They divided 6 1/2 by 2 and 6.5 by 2. They multiplied with fractions when they dealt with 832 x 1/2, 6 1/2 x 64, and 3 1/4 x 128. The sameness of 6 1/2 and 6.5, and of 3 1/4 and 3.25, came up naturally, too.
The questions children ask are often more developmentally appropriate than those found in textbooks.

How We Foster Children’s Initiative

Following Children’s Leads
We foster children’s initiative obviously by picking up on what they say. It is not possible to follow everything they suggest, but the questions they ask are often more developmentally appropriate than those found in textbooks. Children’s questions are developmentally appropriate because they come out of their level of thinking. The children described earlier were excited and confident about trying 832 x 1/2, 6 1/2 x 64, and 3 1/4 x 128.

When a problem is truly theirs, children mobilize everything they know to try to solve it.

Avoiding Algorithms
The second way in which we foster children’s initiative is by refraining from teaching conventional algorithms and, instead, encouraging children to invent their own procedures for solving problems. Most of the children in the class described earlier had been taught to use algorithms, but some had been encouraged to do their own thinking in third grade. Third graders who are not taught conventional algorithms can generally invent many more procedures than fourth graders who have been required to use these rules. As explained in Kamii (1994) with extensive evidence, the teaching of algorithms is harmful for two reasons: (a) Algorithms force children to give up their own thinking, and (b) they “unteach” place value and therefore hinder children’s development of number sense. Each of these reasons is elaborated below. Other evidence of the harmful effects of algorithms can be found in Mack (1990), Narode, Board, and Davenport (1993), and McNeal (1995).

Algorithms force children to give up their own thinking. When children are not taught any algorithms and are, instead, encouraged to invent their own procedures, their thinking proceeds in a very different way from the algorithms. In addition, subtraction, and multiplication, the conventional algorithms go from right to left (from the ones column to the tens column, and so on). However, children’s initial inventions always go from left to right, as can be seen in the examples in Figure 9.

It is clear from the preceding examples that when children are made to use algorithms, they must give up their own ways of thinking. Because a compromise is not possible between going from left to right and going from...
Algorithms force children to give up their own thinking.

right to left, children obey teachers rather than thinking in their own way. Children who have given up their own thinking cannot be expected to have initiative.

The only children who have not been crippled by conventional algorithms are the brightest, most advanced minority in each class, who could make sense of the algorithms. The others, the great majority, could not understand the reason underlying each algorithm, and therefore learned only the steps to perform. Once they have become successful at using algorithms, it is extremely and surprisingly difficult to get children to unlearn them. While computers can be unprogrammed with the touch of a button, children who have become algorithmic robots are extremely hard to unprogram. This difficulty was pointed out more than a dozen years ago by Madell (1985).

Algorithms “unteach” place value and hinder children’s development of number sense. When children use the algorithm to solve problems such as:

\[ \begin{align*}
876 \\
+345
\end{align*} \]

they unlearn place value by thinking and saying, for example, “Six and five is eleven. Put the one down and carry one (or ten). One and seven and four is twelve. Put down the two and carry the one (or ten). One and eight and three is twelve.” The algorithm is convenient for adults, who already know place value. For children, who have a tendency to think about every column as ones, however, the algorithm serves to reinforce this weakness.

By contrast, if children are encouraged to invent their own procedures, they think and say, “Eight hundred and three hundred is one thousand and one hundred. Seventy and forty is one hundred and ten; so that’s one thousand two hundred and ten. Six and five is eleven; so I put them together, and the answer is one thousand two hundred twenty-one.” The children who are encouraged to invent their own procedures thus strengthen their knowledge of place value by having to use it.

When the teacher described earlier listed all the answers the children had gotten, two of them were 308 and 564. These were answers given by children who had learned to use algorithms. Those who do their own thinking seldom get such unreasonable answers because they usually reason that 50 added eight times or 50 x 8 equals 400. The answers of 308 and 564 nevertheless represented enormous progress compared to the 4016 that some children gave at the beginning of the year. Children who do not use conventional algorithms simply do not make such outlandish place-value errors.

After listing all the answers given by children, the teacher asked for comments about solutions that seemed reasonable or unreasonable. The great majority of children who use algorithms have no opinion and are, therefore, passive in such a situation, especially at the beginning of the year. For them, the question of “reasonableness” has no meaning because math has never made sense anyway and has never been reasonable.

Refrain From Judging—Promote Discussion

A third way in which we promote children’s initiative is by refraining from saying that an answer is correct or incorrect and, instead, encouraging children to agree or disagree among themselves.

When the teacher decrees that an answer is correct, all thinking and all initiative stop. If, on the other hand, the teacher does not say:

Children will inevitably reach the truth if they debate long enough because, in logico-mathematical knowledge, relationships are never arbitrary.
that an answer is correct or incorrect, children evaluate each other’s ideas until they reach agreement. In the logico-mathematical realm (explained in Kamii, 1994), children will inevitably reach the truth if they debate long enough because, in logico-mathematical knowledge, relationships are never arbitrary.

In Conclusion
The three principles we advocate—following children’s lead, not teaching algorithms, and not saying that an answer is correct or incorrect—are the opposite of the traditional approach to teaching mathematics. In the traditional approach, each topic, such as the multiplication of fractions, is introduced by the teacher. The teacher then shows the students how to get answers and assigns similar exercises. The correctness of each answer is then judged by the teacher (or by a computer nowadays). This approach is rooted in the belief that mathematics is a set of rules, skills, and concepts to be learned by internalization from the environment. However, Piaget’s constructivism has shown with more than 50 years of scientific research that children acquire logico-mathematical knowledge by constructing it from the inside, in interaction with the environment.

When we understand how children learn mathematics, our approach to teaching changes drastically. Reform in mathematics education no longer means doing better what we have been doing for centuries. It is time to go beyond “helping” children in well-intentioned ways that are in reality harmful to them. □

References


Constance Kamii is Professor of Education at the University of Alabama-Birmingham. Michelle Pritchett and Kristi Nelson are teachers at Trace Crossings Elementary School in Hoover, Alabama.
Learning Together
Kathleen Martin

Teacher educator
Kathleen Martin explains how to use Piagetian tasks to help prospective teachers think differently about how children learn.

Rethinking Teaching
Prospective teachers are a product of years of "being told and telling back." Thus, their own lived experience of school inclines them to view teaching as telling. This view is implicit in the way they think about their own learning and, consequently, about the learning of the children they will teach. The challenge, then, is to break this vicious cycle and to create environments where prospective teachers can themselves experience the initial surprise that characterized Piaget's findings. We tend to forget the early skepticism and the voluminous replication experiments that surrounded Piaget's work.

Initially Piaget's tasks with children were repeated over and over because the findings were so astounding. A child who could count to a hundred did not realize that ten pebbles positioned closely together were the same ten pebbles positioned further apart. A child who insisted on his fair share of liquid refreshment did not realize that the quantity of liquid remained constant when poured into a container of a different shape (Gallagher & Reid, 1981). Based on everyday observations of children, not even Piaget was prepared for his discovery that number and quantity are not natural intuitions for children (Duckworth, 1979).

Precisely because the results are so surprising, the Piagetian conservation tasks constitute a condition for learning among those who observe children engaged in the tasks. In our studies of prospective teachers, we have noted that they are as surprised by the responses of the children to the Piagetian tasks as were early experimenters. We have further noted that the surprise of the prospective teachers is itself a condition for their own learning about children's thinking. The surprise and ensuing dialogue can give rise to transformations in thinking because the tacit understandings of the prospective teachers are brought to consciousness, where they can be considered and changed as needed.

In our work with prospective teachers, we have been using two Piagetian tasks, one that illustrates conservation of area and another that illustrates conservation of volume in a solid. The surprise (and subsequent reflection that the activities elicit) help prospective teachers recognize both the complexity of the mathematical construction of conservation of area and conservation of volume as well as the complex reasoning involved in these constructions. The tasks also help prospective teachers to under-

The research reported in this paper was supported by grants from the Exxon Education Foundation and the National Science Foundation. The views expressed do not necessarily express those of either Foundation.
stand Piaget’s important distinction between empirical experience (which draws its information from the objects themselves) and logical abstraction (which proceeds from the child’s actions and operations and leads to reorganization of thought). These understandings have implications for teaching that extend far beyond the teaching of mathematics.

**Having a “Neutral Zone”**

Our work with prospective teachers occurs in a 2,200 square-foot learning laboratory called Hands On Science, which is located in the Fort Worth Museum of Science and History. University classes in science and mathematics education are taught at the Museum. The undergraduates in the teacher preparation program work with school children who are scheduled to participate in events in the Lab. The Piagetian tasks are among such events.

We refer to the Lab as “the neutral zone.” Practicing classroom teachers and prospective teachers, university professors, and museum educators gather here without institutional impediments. The primary purpose of the Lab is to seek a better understanding of the conditions that promote children’s learning of science and mathematics. Standardized curriculum, instructional methodologies, and operating procedures are avoided in favor of the meanderings and surprises characteristic of natural learning. Instead of trying to take students somewhere, as in the tradition of didactic teaching, the prospective teachers are encouraged to watch where children go intellectually and to observe how they get there. The focus is on constructing learning environments that are compelling to children, environments that capture their interest and challenge their thinking.

The atmosphere of the Lab reflects the belief that children, like all human beings, act on and give meaning to their world. The philosophy of the Lab also espouses Seymour Papert’s (1980) notion that children need “objects-to-think-with” (p. 11). Thus, the Lab is filled with objects that invite handling, particularly objects that issue the challenge to build. The many building materials accessible in the Lab encourage a focus on spatial relationships.

**Watching Children**

Attention to conditions for learning is the central focus of our work in the Lab. The Piagetian conservation tasks operate as initial conditions for engaging children in building structures where their reasoning related to conservation of area and volume can be observed by prospective teachers. The tasks are fundamentally the same as those described by Piaget, Inhelder, and Szeminska (1960) in *The Child’s Conception of Geometry*.

While the building process is the condition for engaging the children, “kid watching” is the condition for engaging the prospective teachers. Prior to actually working with children, the prospective teachers view a video in which they observe children from ages four to 12 years performing the tasks in the presence of a clinical interviewer and listen to Piaget’s interpretation of the children’s actions. They also perform the tasks themselves and discuss their roles as observer and interviewer in this context. The prospective teachers are expected to write papers detailing the actions and words of the children with whom they work and their own actions and words during observation of the tasks. The prospective teachers are then to interpret their observations in terms of the children’s thinking.

The prospective teachers whose actions and descriptions are referenced in this research observed second-grade students for the conservation-of-area task and third-grade students for the conservation-of-volume task. Within these age groups, the prospective teachers encountered children at a variety of different levels of maturity and experience. The wide variation among children of the same age group compounded the prospective teachers’ surprise at the things that the children said and did.
Seeing Complexity in the Seemingly Simple

Both Piagetian tasks evoke in prospective teachers an awareness of the tension that children experience between empirical experience and logical abstraction. In the conservation-of-area task, the children are shown two 8 and 1/2 by 11 inch sheets of green paper that represent pasture land. A small replica of a grazing animal such as a cow or sheep is placed on each pasture. Barns are then placed alternately on the pastures, one by one. On one pasture, the barns are lined up adjacent to one another on a pasture boundary; on the other, the barns are scattered throughout the interior of the pasture. Prospective teachers continue the placement of barns until each pasture has 18. During the process of positioning the barns, the prospective teachers ask the children which animal has the most grass to eat and why. During the course of the activity, many of the children say that the pasture with the adjacent barns on the boundary has more grass for the animal because the area appears larger; on the other pasture, the area is broken up or segmented by the barn placements and does not appear to be as big.

While the prospective teachers are initially surprised that some children would see the segmented pasture as having less grass, they are able to recognize the power of visual perception and the inclination of some children to favor such. However, the prospective teachers are considerably less astute at interpreting the understanding of children who are experiencing conflict between their empirical and logical ways of knowing. Piaget (1995) referred to these children as “transitional” and pointed to the self-regulating process through which thought develops:

... in the construction of any operational or preoperational structure, a subject goes through much trial and error and many regulations which involve in a large part self-regulation. Self-regulations are the very nature of equilibration. These self-regulations come into play at all levels of cognition, including the very lowest level of perception. (p. 838)

The prospective teachers often encountered transitional children within the context of the Piagetian conservation tasks. For example, many children count the number of barns on each pasture each time and will say “each pasture has the same number of barns and so each cow has the same amount of grass to eat,” or “this pasture has one less barn and so this cow has more grass to eat.” One child consistently formulated her argument this way until each pasture had 18 barns. The prospective teacher then told her to look very closely at the pastures one more time and tell her which cow had the most grass to eat. The child immediately responded that the cow on the pasture with adjacent barns had more to eat because there was more space; the cow on the other pasture had only little spaces of grass to eat. The prospective teacher was completely taken aback by what she deemed “a change of mind” on the part of the child and was flustered because she had no way of accounting for the “change of mind.”

The prospective teacher had to learn that the child did not “change her mind” as much as she changed what it was that she was focusing upon to make up her mind. The intervention of the prospective teacher undoubtedly helped shift that focus. Initially, the child was focusing on counting. The child’s reasoning was concentrated on counting, and she realized that it made no difference if the 18 barns were in a straight line on the pasture boundary or scattered around the pasture interior; the number of barns was not contingent upon the arrangement of the barns. However, conservation of number is not synonymous with conservation of area. The reasoning related to conservation of area requires the child to correlate the number of barns with the area of grass covered by each barn. When this particular child shifted her focus from the number of barns to the amount of grass, she immediately “changed her mind.” The child
was able to reason that the number of the barns was not changed by their rearrangement. When she focused on the number of barns, she responded that the cows had the same amount of grass to eat. When, however, she shifted to a spatial focus and looked at the amounts of visible green space, she was not able to reason that the area of grass under the barns was the same, even though it was distributed differently.

Piaget (1995) describes the conflicts that transitional children can experience between what he terms “subsystems” of knowledge:

These subsystems can present conflicts themselves. For example, it is possible to have conflicts between a subsystem dealing with logical-mathematical operations (classifications, seriation, number construction, etc.) and another subsystem dealing with spatial operations (length, area, etc.). (p. 839)

When the child bases her judgment on the number of barns, she makes one judgment of quantity. When she bases her judgment on the area of grass, she makes a different judgment. The second judgment appears contradictory to the first. According to Piaget, the child cannot yet coordinate the subsystem related to number and the one related to area and thereby establish equilibration.

Several children evidenced reasoning related to conservation of area. One child explained that you could cut all of the grass around the scattered barns and put it in bags, and then cut all of the grass on the other pasture and put it in bags, and that you would have the same number of bags of grass. Another child explained that if you could take the grass under each of the scattered barns and line it up in rows, it would give you the same amount of grass as covered by the adjacent barns. These two children were able to reason through the relationship between the number of barns and the amount of grass under the barns. They spontaneously provided these explanations when the prospective teachers suggested that it was difficult to “see” how the animals could have the same amount of grass to eat, when it “looked like” the pasture with the scattered barns had less grass available. The children gave their explanations in a somewhat condescending tone because they thought that the prospective teachers should certainly be able to see the relationship that was so obvious to them. Piaget refers to this sense of the obviousness of the relationship as “necessity.” These children experienced no conflict between quantity judged in terms of the number of barns and quantity judged in terms of the area of grass.

Just as the conservation-of-area task provides an opportunity for prospective teachers to see the complexity that underlies the mathematical concept, so it provides insight into the complexity of learning the concept. Not only did the task make visible how children reasoned differently depending upon whether they focused on number or space, but it also helped them understand the challenge that the children faced in learning to simultaneously hold and relate the two perspectives.

Many of the prospective teachers who have worked with children on the conservation-of-area task have not been able to see beyond the procedural simplicity of the task to the underlying conceptual complexity. To some extent, the problem lies in the understanding of the prospective teachers themselves. Their own connection of the number of barns with the area of grass covered by the barns is so taken for granted and the relationship has such a “necessity” about it
that they are not able to recognize the intellectual struggle required of the children to achieve the connection. Those children who manifest considerable confidence in the conservation of number and, consequently, are insistent that the animals have the same amount of grass to eat are able to persuade the prospective teachers that they really have the concept. Their confidence is compelling, and the prospective teachers tend to assume that these children know more than they actually do because they “answer correctly.” This evidences the tendency of the prospective teachers to focus on “answers” rather than the conception underlying the answers. Such a tendency is easily evoked because the behaviorist psychology that has forced this interpretation of their own learning inclines them to interpret children’s learning in simple causal terms. Thus, the equality of the number of barns “causes” the equality of area of grass. The more complex relationship revealed through a Piagetian interpretation is eschewed by the simpler behaviorist causal relationship.

Those moments when children have the greatest potential for learning are characterized by conflicts, gaps, and contradictions. In the course of observing children engaged in Piagetian conservation tasks, prospective teachers are afforded opportunities to see children experiencing these learning moments. However, such moments are uncomfortable and, just as the prospective teachers are inclined to want to reduce disturbances experienced by the children by telling or showing them “correct answers,” so they are inclined to reduce their own disturbances by evoking images of teaching as a didactic relationship and of curriculum as a controlling context.

**Cultivating Natural Powers of Organization**

The conservation-of-volume task is a particularly viable learning environment in which prospective teachers are able to observe and reflect upon the natural organizational powers of children. The procedure for the conservation-of-volume task again parallels that described in *The Child’s Conception of Geometry* (Piaget, Inhelder, & Szeminska, 1960). The child is shown a solid block measuring 4 units in height with a square base of 3 x 3 units. Thus, the volume is 36 cubic units. The child is told that the solid block is a hotel and then asked to build another hotel that has exactly as much room and to build it on a base with an area that differs from the base of the original solid hotel (e.g., 1 x 3 units, 2 x 3 units, or 4 x 3 units). The child is provided cubes with which to build that are 1 cubic unit. The child’s problem is to construct a building with the same volume as the first building while changing the form to comply with one of the given bases. Construction must occur on an “island” base that is surrounded by water.

Conservation of volume is a more complex concept than that of conservation of area; therefore, prospective teachers tend to be more challenged to understand how young children think about the concept. Their formulaic knowledge of volume as length times width times height tends to make them think of the “answer” in those terms. A hotel that has a base of 3 x 3 units and a height of 4 units is then thought of as having a volume of 3 x 3 x 4 units. Most of the prospective teachers recognize that the 36 cubes must be conserved when building on a different size base.
Their tendency, however, is to tell the children to build a replica of the solid hotel and then lead them to use those same 36 cubes when building on any size base. The prospective teachers generally fail to realize that the children in such instances are conserving the number of cubes but do not necessarily correlate the number of cubes with the volume that they occupy. For example, one child agreed that the 36 cubes that he used to make a replica structure would have to be used on a 2 x 3 base and proceeded to build a six-floor hotel. When, however, he built a hotel on a 1 x 3 unit base, he stopped at 11 floors. He said that he knew the hotel should have 36 blocks but that it was “just too tall” and that if he added more blocks, it would have more room than the original hotel. Even though the prospective teacher led him through a logic whereby he agreed that both hotels should be made from 36 cubes, he refused to add another floor to his 1 x 3 base hotel because it would be bigger and have more room. His empirical knowledge prevailed.

Many of the prospective teachers were quite perplexed if children were inclined to stop building when their hotel was the same height as the solid structure. This was particularly true when the base was 1 x 3 units and the children built the hotel with only four floors to match the height of the original solid hotel. While most of the children recognized that their constructed hotel did not have the same amount of room, many were at a loss about how to make them the same. They insisted that they had to be able to build in the water in order to replicate the solid hotel and thus build a structure with the same amount of room. What was even more surprising for the prospective teachers were the responses of the children when the solid hotel was turned on its side so that its base changed from 3 x 3 units to 3 x 4 units. Since the transformed hotel was then only 3 units high, a number of the children proceeded to remove a floor from the hotel that they had constructed in order to make the hotels the same height. This transformation process made it abundantly clear that the children who removed a floor from their hotels were focused on a single dimension, the height. When the solid hotel was turned on its side, some children even took all the cubes off of their island and then rebuilt their structure so that it coincided with the height of the solid hotel. The transformed hotel, from its upright position to its sideways position, was perceived as a different hotel by those children who focused on height.

Because the actions of children engaged in the conservation-of-volume task offer prospective teachers so many surprises, the opportunity is provided to unsettle their notions of learning and teaching and subsequently encourage reconsideration. The “delivery system” notion of instruction, to which most prospective teachers were themselves subjected and which continues to prevail in schools, sees curriculum as logically ordering knowledge which is then transferred directly to the student through instruction. Within this cause-and-effect framework, curriculum organizes and controls thinking. A Piagetian framework emphasizes instead the natural organizational powers of the child, the openness of the cognitive system, and the transformative nature of thought through self-organization:

Structures of thought made possible by the previous reorganization lead to conflicts, gaps and contradictions which cannot be accommodated within the present system. This leads to a need to create new structures, coordinations and operations which will accommodate...
these contents without the conflicts, contradictions, and gaps that existed in the previous structure. This process of equilibration and disequilibration is responsible for the growth of human thought, according to Piaget and Garcia. (Reynolds, 1997)

While the Piagetian tasks are designed to reveal the child’s natural organizational powers, their visibility to prospective teachers is not guaranteed. The prospective teacher who is blinded by a view of curriculum as controlling what the child is to learn tends to bear the belief that teaching is simply a matter of telling or showing the child: “I believe that if someone showed her what was missing from her hotel and how she could fix it, she would learn to grasp that concept.” The prospective teacher who arrived at this interpretation had been observing a child who was focused on height. The child had built a hotel that was 4 units high on a 3 x 1 island base. The height of her hotel corresponded to that of the original solid hotel. Although the child knew that the two structures did not have the same amount of room, she was unable to escape her focus on height long enough to discern a way to make the volume the same without building in the water. When the 3 x 3 x 4 hotel was turned from its 3 x 3 base to a 3 x 4 base, the child removed one floor of her constructed hotel to make the height of her hotel equivalent to that of the reoriented solid hotel. Yet, the prospective teacher wrote that she felt the child was “on the verge of learning the third dimension because she seemed to know that something was missing from her hotel.” This interpretation is characteristic of those rendered by prospective teachers who have difficulty making sense of children’s actions as they relate to children’s thinking.

In the illustration cited, the prospective teacher probed the child in an effort to lead the child to consider building her own hotel higher to compensate for the “wider” solid hotel. She writes that the child “was stumped, so she took down the hotel and proceeded to build another, which ended up looking exactly like the last one.” This action, together with the child’s earlier response to a change in the orientation of the hotel, provided the prospective teacher with considerable evidence of the child’s focus on height. She had no evidence that the child had even a qualitative sense that she could build her own hotel “taller” because the solid hotel was “wider.” Still the prospective teacher considered that the child was “on the verge of learning” and probably would learn the concept “if someone showed her what was missing.” This prospective teacher, like others, seemed to have difficulty focusing on the organizational powers of the child evidenced in her actions. The conservation-of-volume task, then, was seen as a curricular context for teaching the child the concept, rather than as a condition for making visible the child’s thinking about the concept.

Educating Prospective Teachers

The Piagetian perspective challenges teacher preparation programs to establish conditions that enable prospective teachers to see and think differently. If prospective teachers are to promote the natural organizational powers of children—powers inherent in a dynamic, self-regulating cognitive system—then the prospective teachers themselves must engage in the use of such powers. The Piagetian conservation tasks seem to offer a context for this essential learning.

References

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Marilyn Watson: Building Ethical Understanding and Commitment: What Does It Take? What Does It Look Like?

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Creating Math Games That Support Young Children in Developing Number Sense.
  Alice Wakefield, Old Dominion University, Norfolk, VA
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The Constructivist College Classroom: Hard-Hat Required.
  Alice Tomasini, Elaine Chin, University Center for Teacher Education, Cal Poly, San Luis Obispo, CA

Making the Most of the Classroom Mosaic: Constructivist Teaching in Inclusive Classrooms.
  Margy Gray, Fontbonne College, St. Louis, MO; Bruce Marlowe, Johnson State College, Johnson, VT; Marilyn Page, Johnson State College

Examining Science Education-As-It-Is: Considering Science Education-As-It-Could-Be.
  Dewey Dykstra, Jr., Boise State University, Boise, ID

Constructing Explorations for Science, History, Literature, and Life.
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Class Meetings: A Model That Enables Children With Special Needs to Develop Empathy and Participate Fully in the Classroom.
  Mona Halaby, Park Day School, Oakland, CA; Jill Alban, San Francisco and Oakland Unified School District

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