The Constructivist
Fall 2007
Vol. 18, No. 1
ISSN 1091-4072

For a Holistic Framework of Constructivist-Generative Teaching (CGT)

Octavian Lecca
DELFO Academy
Montreal

Doina Lecca
Concordia University
Montreal

It's not the strongest species that survive, nor the most intelligent, but the most responsive to change.

- *Charles Darwin*

Abstract

This article is an attempt to bring under the unifying umbrella of a pragmatic model (Lecca D., 1996), different views on education based on psychology, mathematics, linguistics, biology, and system theory. Considering *responsiveness to change* the main skill of dealing with the challenges of the 3rd millennium, the authors define *self-awareness* as the generative goal of education. Seen from this perspective, the surprising cohabitation between constructivism and behaviorism is not only possible but also necessary since behaviorist teaching is meant for the part of the world to which we must adapt, while constructivist teaching enables individuals to change the world. The theoretical model is exemplified by several applications in mathematics.

Introduction

Sometimes it happens that an apparently insignificant event may be a turning point in someone's way of thinking. This happened to me when at a party an old teacher, asked to give an example of an unusual thing which occurred in his career, had the following story:

> I was teaching geometry, and I asked my students what is the difference between a right angle triangle and an isosceles one. They answered correctly that the right angle triangle has a 90 degrees angle and that the isosceles triangle has either two equal sides or two equal angles. "But they also have different tastes" replied another student. The whole class burst into laughter. The student looked puzzled. He hadn't said this to make the class laugh! It was proven later that this student associated taste not only with shapes but also with words. The end of the story was that after a series of medical tests, the student's case was dropped because it wasn't considered to be worth "serious" scientific investigation.

We take this as a clear case of ignoring diversity and trying to convert it into uniformity. What is the role of teaching after all? Is it to train an ideal individual for a given society without envisaging future scenarios such as the change of the individual or his/her initiating a change of society? Should or could our teaching prepare individuals to be able to cope with both these situations? It should be kept in mind that confronted with an unprecedented rate of change, societies need a different way of educating their new generations. The young employee who has just finished school should be able to deal with an avalanche of new products, new standards and new cultures and strategies invading the global market. At the moment, constructivism appears to offer a solution to this problem.

Analysis of CGT

Why is the constructivist approach difficult to accept?

It is generally accepted that the need for learning springs ultimately from the need for adapting to one's own environment, be it family, society, the earth or the universe itself. What school teaches students about the outside world is in fact how to build their adapting strategies to existing mini-worlds.

Still, school is not doing enough in terms of teaching human beings to make use of and enhance their precious innate gift of building new worlds. How can we possibly teach students this complex skill, which should necessarily develop simultaneously with a process of self-awareness?

At this stage, it would be important to find out how profound the differences yielding human diversity are and, in relation to this, how significant it is to adopt uniform versus diverse teaching. The latter would imply having access to every student's mental processes prior to choosing the appropriate teaching method, which, on a large scale, seems to be an impossible task. So we are left with two options: either we define the model of an average human being and we generalize it to everybody, or, we give students tools to discover themselves and, subsequently, the self- teaching methods appropriate to them.

Thus we have actually defined in simple terms the two existing main teaching directions: *behaviorism* and *constructivism*. Opinions vary as to whether these directions are mutually exclusive or they complete each other. We believe that the debate between accepting one method or another or accepting a combination of both depends on the answer to the question that we haven't answered yet: How deep is human beings' diversity in a given society and how significant is it for the teaching process? In other words, it would be useful to know the border between accepting and not accepting diversity.

An example of a form of diversity which is sanctioned by already established society rules is crossing the street when the light is red. In schools teachers often require and evaluate newly acquired behaviorist skills such as learning by heart the world's capitals or the multiplication table. We always teach children not to touch an electric wire before teaching them *what* electricity actually is. A similar sort of "behaviorist teaching" applies also to the animal world: a young lion is taught that it shouldn't attack other animals in times of drought near the source of water, but is helped to develop its own hunting skills in a "constructivist way" in a real hunting situation.

In other words, behaviourism defines a type of human being perfectly matching society by complying with the existing rules of society. Exceptions are not allowed but only tolerated until the individual "is back again on the right track". The main goal of behaviourist education is "teaching" individuals how to be "successful" in a well-defined society. Priority is given to class management, extended sometimes to what we would call "thought management". On the other hand, constructivism switches the focus from teaching to learning. Knowledge is no longer "transferred" by the teacher to the student, but it

must be constructed by the learner based on his/her ability to match events to possible worlds[1] of interaction.

Over the years, the concept of education has evolved from the paradigm "individuals make a society" to "society is made of individuals". Since in the latter case the emphasis is on society, individuals are afraid of seeing their reasoning distancing itself from the rigid social model they happen to be in. *The fear of being different*, arguably one of the oldest social syndromes, might be the main cause why the constructivist approach is not easily adopted. As constructivist teaching encourages the development of the individual's own reasoning, there is a great chance of deviation from the norm, while the prescriptive rules of behaviourism teach individuals how to "tame" their reasoning and make it "well-behaved".

Accepting the constructivist approach means in fact accepting that we are all different. This is a difficult threshold to cross as most societies promote a formal, "ideal" model, in keeping with their traditions as well as their national and cultural taboos.

On Constructivist Education

Until not long ago social changes occurring during one generation were not as noticeable as they are today. Nowadays routine tasks are left to robots while head hunters fiercely compete to recruit competent staff able to manage the change. Constructivism is seen by many governments as the solution to their educational reforms: constructivist teaching is giving students the knowledge of analysis/decision tools and how to use them and learning is how every student applies those tools to a particular context of which a necessary part is his/her own personality. In fact, constructivism emphasizes **the knowledge of self**, which implies the teacher's "need to construct a hypothetical model of the particular conceptual worlds of the students they are facing" (Twomey, C., 2005, p. 7).

Therefore, a constructivist school should provide students with **modeling tools** (to find out inner/outer worlds or to imagine possible worlds), **matching tools** (to be able to link those worlds and negotiate them with others), and **strategic tools** (to be able to apply the modeling and world matching tools to accomplish a goal in a given world). School should also coach students how to apply those tools to their own personality and use them appropriately.

In contrast with the traditional school, in a constructivist environment, the student is obliged to discover his own truths about the outside world and oneself - a challenging task for all the actors involved.

Pragmatics and the need for reference

Unlike sciences whose domain is clearly defined, education still escapes a clear definition. We need to define the ultimate goal of education in order to identify a principle-based theoretical framework of teaching. There have been many attempts to define such a goal. However, many of them are at the societal level, and not at the individual level. If we agree that education is directed towards individuals' self-development and not towards turning them into obedient robots for serving societies, we must look for an individual generative characteristic which education could foster.

We believe that such a characteristic is **autonomy** as proposed by Kamii (2004, p. 49). However, we think that Kamii's meaning of autonomy – "being governed by ourselves" (p. 45) - could be further

[1] This concept will be defined further in this article.

clarified. We believe that autonomy can be reached only if the individual knows his/ her own inner as well as outer worlds and does not rely on an alien model. Moreover, if we consider *the aim of education* as the development of *individual's own generative reasoning*, we feel we could cast more light on what successful education means.

To begin with, as the biological construction of our outside world is based on generative principles (Lecca, D., Lecca, O., 2005), we think that constructivism should be taught in a generative way. Embarking on this task, we resorted to an "outsider's view" to assess the role of education - *pragmatics* - a highly interdisciplinary, relatively recent branch of linguistics. Its emphasis on *meaning construction in context*, of which a necessary part are the *mental processes of interlocutors* has earned pragmatics this right.

Secondly, from a pragmatics point of view, truth is context dependent, which emphasizes again the need for a generative approach. Let's say that at sunset, a human, a dog, a bat and a snake see a mouse. Do they have the same perspective? Certainly not. Each of them "sees" the mouse from a different perspective: motivation, characteristics, etc. and still, all are right. However, there is a common goal that would give them a unified perspective: to minimize conflict and maximize cooperation with the outside world (Lecca, D., Lecca, O., 2005).

By using context as the source of interpreting the truth, pragmatics shows that every individual has his/her own truth and that the context should be as wide as possible for the truth to be as accurate as possible. If Piaget tried to explain education universals starting from reality, pragmatics tries to explain reality starting from universals of communication. Since communication can only be successful when both interlocutors reach a common ground, we claim that self-awareness, as a prerequisite of constructivist teaching, occurs where the principles of the outer world and of one's inner world will match.

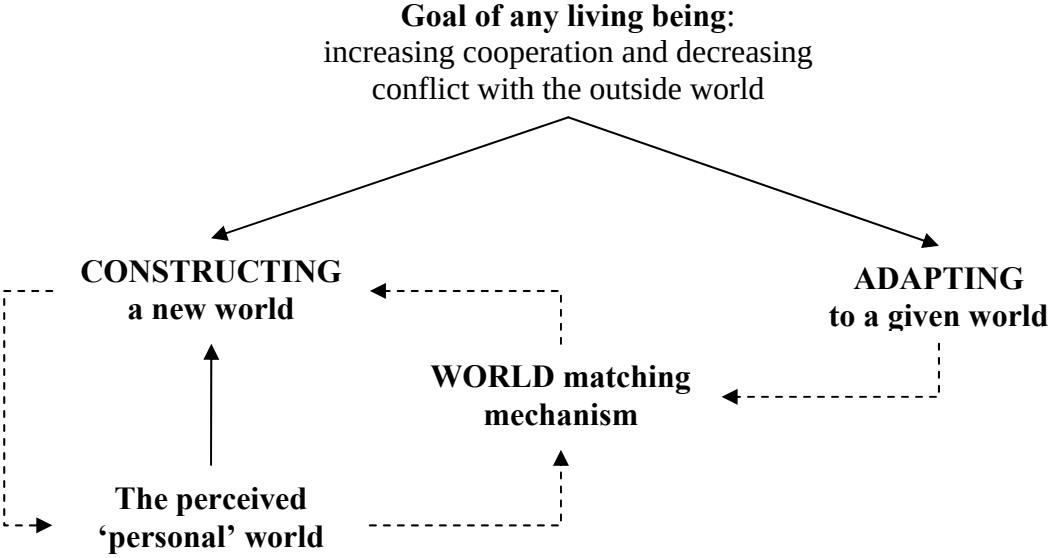

Fig. 1

This way, modern man will not only adapt to the part of the real world which is consistent with his/her beliefs but will also try to change the part of the real world where the two sets of principles conflict with each other in order to feel "at home". Any system of education should be able to develop such a *WORLD matching mechanism* (see Fig. 1), which is the prerequisite of the individual's autonomy.

If adapting to an existing world seems to be an "easy" task, as the world is known or, at least it could be investigated with our biological tools, constructing a new world to which the individual should already be adapted, has, besides the construction effort, an additional hurdle to overcome: the individual should a priori "know" his/her own world. Unfortunately, we don't possess the biological tools for self-investigation.[2] This is why, we usually start with an improper or false representation of our own world, either "borrowed" or partially experimented from our previous interaction with the outside world. During the construction process and through a *world matching mechanism*, we discover new facets of our own world, which naturally trigger changes in our own representation of ourselves as well as in the world we want to construct and to which we are supposed to be fully adapted.

In this respect, if we were to define behaviorism and constructivism, we would say that behaviorism corresponds to the WORLD-adapting mechanism and constructivism is related to the WORLD-changing mechanism. Thus, any individual has an ideal WORLD-model, which is inevitably confronted with the existing WORLD that he/she would claim to belong to. A winning solution is to compromise and to accept a commonly negotiated world.

The World as Context

According to Rescher (1974, p. 78), "possible worlds are collections of compossible sets of possible individuals duly combined with one another". If we consider a *generative* principle lying at the basis of our rationality, we may see possible worlds interrelated like an infinite set of Russian dolls where opening the door of one system leads inevitably to another one. According to Jaszczolt (2002) the individual's mental processes are like a sieve through which we sort possible worlds into those the sieve retains and those it discards.

We start from the assumption that an individual has a dual model, represented by both his/her real world as well as by his/her alleged membership to a specific social world (see Fig. 2).

The *real* specification representing the individual's *real world* (*W'r*) is more often than not perceived as distorted and/or incomplete. In fact, at a given moment, the other actors with whom the individual comes in contact have only access to this distorted/incomplete image of his/her *real world*, which we called (Wr). In the process of interaction, (Wr) is likely to be clarified, but it could also be further distorted.

Wr = the "real" world (perceived by the individual as his own world)

W'r = the "true" real world (of which the individual is not necessarily conscious)

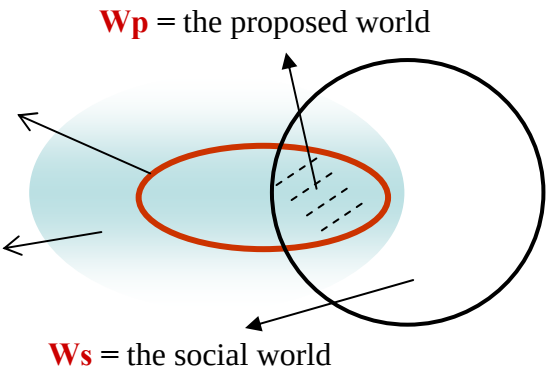

Fig 2

But why is the individual's real world so difficult to perceive?

[2] The main goal of psychoanalysis is to develop self-investigation tools.

First, the human being has a double macro-strategy (see Fig. 1) **- trying to adapt to a World the way he/she sees it** (adaptation strategy) **and trying to modify this world the way he/she sees himself** (changing strategy). Which of the two strategies prevails and what is the right balance between them is given by each individual's record of interaction and by the ratio of his/her successful versus failing communicative acts, which is best perceived in the negotiation process. Brown & Levinson's (1987) *Model Person* (MP) – "rational agent with face", is a useful concept not only in presenting a more accurate *self* but also in assessing the fluctuating balance between adapting and changing strategies in the above mentioned process.

A winning strategy would be to make interlocutors accept one's world. For an effective communication process, interlocutors need to have a correct reciprocal representation of their worlds, which allows them to propose a common world based on a mutually acceptable balance between adaptation and change.

Furthermore, in order to have a real winning strategy, an important prerequisite would be that the speaker could not only build a correct representation of the hearer, but also have an accurate perception of his own representation in the hearer's mind (e.g. We are very interested to know what our students "think" about us). If we do not take into consideration this process in education, the message fails or it is wrongly interpreted. A schematization of the process of building up a winning strategy would be formulated as in Fig. 3:

I want her to do (X), which is in my interest. The way I know her, she believes (Y) about me. If I send her the message (Z), I will make her believe that doing (X) is in her best interest.

Fig. 3

The concept of *world* – a basic component of our model, cannot disregard important findings in psychoanalysis such as the disturbing coexistence of a *false self* and *a true self* (see Fig. 4), the latter being sometimes dormant until it is provided a medium for growth (Lomas 1987, p. 84). With Freud, we start witnessing the fascinating construction of the s*elf* through communication, a process often blocked by the enforced or deliberate adoption of a *false self*,[3] resulting in apparently well-integrated social chameleons, or intruders who adopt the specifications of the world they find themselves in.

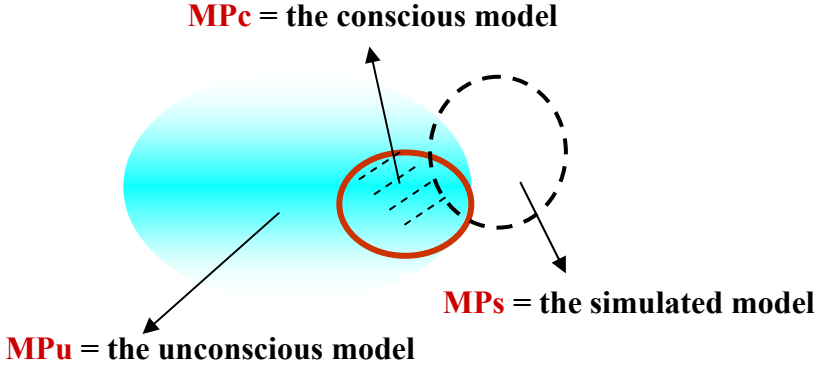

Fig. 4

[3] We replaced the concept of *self* by *Model Person* (MP), which we believe is more accurate.

Another breakthrough in psychoanalysis which had an important impact on human communication was the recognition that "our perceptions are colored by an inner world of which we are largely unaware" (Lomas 1987, p. 3).

What is then the connection between this complex communicative model and autonomy as the goal of constructivist teaching? We believe that autonomy is reached anytime the individual has a fair representation of his/her own worlds and of interlocutors' worlds. We also believe that it could be measured by an increased feeling of responsibility.[4] If one "knows" oneself, one is necessarily responsible for his/her own actions.

The Individual's Pragmatic Model (PM) we put forward consists of an MP (Model Person) about oneself, a SMP (simulated MP) - the model that we try to build in our interlocutors' minds, as well a PMP (perceived MP) - the model that our interlocutors build about ourselves. Based on this model, communication consists of two separate processes, both based on reciprocal modeling (see Fig. 5).

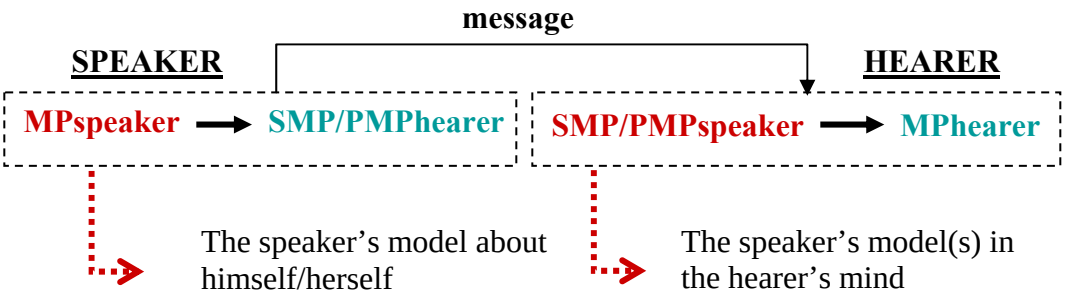

Fig. 5

The individual's simulation mechanism is often hard to detect by less pragmatically competent individuals. Being able to access both the SMP (simulated model person) and the PMP (perceived model person) of one's interlocutor will naturally be an asset which will give someone an upper hand in the negotiation of worlds. In addition to pragmatic competence, elements of psychoanalysis may be required to distinguish between the *conscious, unconscious, and hidden* part of every PMP and SMP.

Returning to the question we asked at the beginning of this article regarding the importance of the differences between human beings, this model demonstrates that even if the tools and methods used are similar, the way of building one's own world and the way of perceiving the outside world are different for every individual.

Furthermore, as modern man acts in artificial worlds created by himself, he may very well find himself in an intruder's position. Also, the "world building process" of communication in general and of teaching in particular has to take into account both production and reception, whose inextricable link is sometimes overlooked.

As suggested earlier, the coherence and efficiency of communication depends to a large extent on interlocutors' accurate *mutual modeling*. At any moment our perceptions might be distorted by our hidden worlds which will determine, sometimes unconsciously, a world shift which is not always predictable or preventable.

[4] According to Larochelle and Bednarz "The constructivist approach creates additional discomfort, so to speak since, by the same token, it reintroduces the notion of responsibility for one's actions" (p. 5).

The world matching mechanism

The individual's world and, therefore, models can be better understood in the light of a growing field of research where the horizon of human analysis has been widened by resorting to supraordinate/larger systems such as the bios.[5] The bios represents the largest system we are "allowed" to consider[6] in an "attempt to develop a biological model of epistemology – a more contemporary framework for a psychological theory of learning" (Twomey, 2005, p. 10).

Assuming once more that the axiom of the biotic system is adaptation to the outside world by diminishing conflict and increasing cooperation with it, we infer that every organism must have a representation of the external world in which changes are triggered by modifications of external conditions/stimuli.[7] We may therefore postulate that: **Learning is not directly triggered by external stimuli, but by the individual's representation of these stimuli, which helps the individual build his/her own unique representation of the outside world.**

At this point, we should distinguish between *theme,* which is an evolutionary direction, a vector describing a possible worlds, and *axiom,* which is an application of the theme and which defines the construction of a specific world. For example, plants and animals, in spite of having incompatible *axioms* underlying their systems, have nevertheless the same *theme*.

Based on the above two concepts, we may now view constructivism in two stages: (1) **meta-constructivism** - a higher level constructivism, in which the "blueprint" of our construction comes to the foreground and (2) **early-constructivism** - a basic level of constructivism generated by the axiom of the "world under construction". Considering both of them as stages of the teaching/learning process as well as considering the generative process of the construction, results in what we believe to be the "holistic" aspect of constructivist teaching and learning.

Early-Constructivism

Adapting to any possible world, which is the requirement of traditional education seems to be insufficient. It may even have negative consequences at the stage of early childhood. "Any new born baby opens the road to selection and to speciation for a sum of possible individuals who may appear and with whom he cannot yet identify. From a biological point of view the new born baby is a 'monad' in which a possible world is trying to appear" (after C Noica, 1986, p. 167). The main requirement of early-constructivism for this age is to teach children how "to see the world" rather than teach them "how the world is".

In early childhood, parents should help children make connections within their direct world in their own way. No constructivist learning/teaching would be possible until this lesson is learned. In Fig. 6, consider 1 the child and 2 the parents: both 1 and 2 follow different logical paths to perceive A, so A has different meanings for 1 and 2.

Moreover, although D is perceived by both parents and children from the same perspective, differences lie in the construction of their perception and are even more difficult to grasp. C has a meaning for the

[5] We have adopted Ludwig von Bertallanfy's definition of system, further developed by the Vienna school within "Die Systemtheorie der Evolution".

[6] If we may claim that we could analyze human reasoning from "outside" by using the "reasoning" of the bios, we can not claim that we can analyze the bios itself since we are not able to place ourselves "outside" it.

[7] "… amphibians lived on land because they chose to and not because they had to" (Taylor, G.R., 1984, 62, referring to the huge changes amphibians had to go through in order to change their marine habitat).

child but it makes no sense to parents. Good efficient teaching implies that the parents help the child build a personal, direct relation to A. Parents should also try to understand C through the child's perspective and not to impose their own perspective. Making children aware that it is their duty to discover their own world is the first constructivist lesson they should be taught.

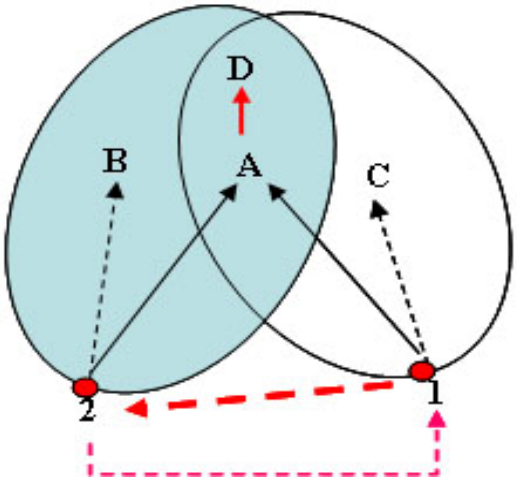

Fig. 6

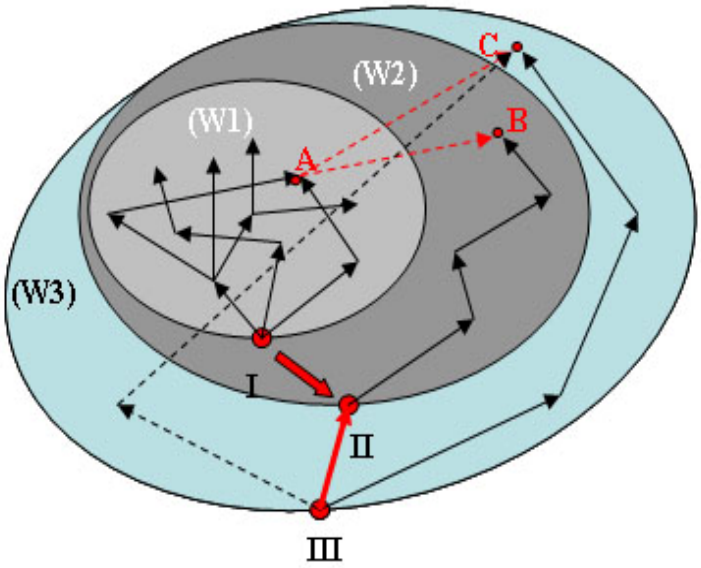

Fig. 7

Meta-constructivism

We would like to further emphasize that constructivism is inherently generative. Its generative feature is essential in linking worlds. In other words, every individual forms his/her own "Russian dolls" style way of connecting existing or newly built worlds. Parents should help children to broaden their world so that event B (Fig. 6) would make sense to them.

Learning how to integrate new events into one's own set of possible worlds based on the child's initial sets of beliefs is the next compulsory step in reaching self-awareness.

In Fig. 7, from the perspective of (W1), A is true and B is false. To prove that B is true, one has to enter (W2) and to find a logical path from II to B, which would mean to perceive the event B as possibly generated by the (W2)'s axiom.[8] There are events which could belong to two or more worlds, which means that there are worlds which share the same events, as is the case of B for (W2) and (W3). It means that for the same event there may be two or more correct interpretations. However, the interpretation generated by the 'wider world' is likely to be more accurate.

Strategic-constructivism

When a new event occurs, we consider that the first step is to place the event within the meta-category of a world, either by generalizing (finding the next wider world) or by particularizing (finding a micro-world). This choice of finding the best *generalization/particularization path* for interpreting an event is determined by the individual's analysis and decision as to where his/her goal could be better fulfilled. This is what we understand by *strategic-constructivism*.

[8] An event in a possible world can only be explained by the truth resulting from the world generating axiom.

Mathematical constructivism

The generative character of constructivism is better seen in mathematics as mathematics "perceives" the world through the "lens" of its basic axioms.

The "lens" of *two parallel lines which never meet* generated the Euclidian geometry, while the "lens" of *two parallel lines which meet at the infinity* generated the Lobachevski geometry. Based on these "truth conditions" mathematics discovers new relations between categories enabling us to refine our (generative) perception of the physical world. Obviously, the "newly discovered truth" is generated by the "initial truth", which is actually the axiom of the broader world we can build.

We keep constructing on predetermined foundations. For example, the foundation of algebra is based on counting. To allow a generalized counting, the world of addition is proposed. In this world, we developed another micro-world – multiplication, seen as repeated addition.

$2 \times 3 = 3 + 3$ and $3a = a + a + a$

Then we continue particularizing by defining the exponential calculations, seen as repeated multiplications, and so on.

$4^2 = 4 \times 4 = 4 + 4 + 4 + 4 = 16$

It depends on us whether we interpret an exponent as belonging to the world of exponents, or belonging to the wider world of multiplication or to the even wider world of addition.

Although mathematical constructivism is usually seen as a particularization path since it defines a new micro-world within the given one, it also means the reverse process of, for instance, subsuming two worlds to a common wider one.

In Fig. 8, if (W1) and (W2) are given, the student is asked to find an encompassing world for the two given worlds, which is generated by O. In such a generalized world the event A could "connect" to any event of (Wo) while C or B are restricted to their own worlds. In this sense, it is recommendable to consider C and B as belonging to (Wo) instead of (W1) or (W2) in order to increase their networking capabilities.

A good example in mathematics is the generalization between the first degree equation and trigonometry.

We notice that the coefficient (a) of the first degree equation : $\mathbf{y = ax + b}$, actually represents a trigonometric ratio - $\mathbf{(\tan \alpha)}$. So we can write:

$\mathbf{y = (\tan \alpha)\, x + b}$ and, as a consequence:

$\mathbf{\alpha = \tan^{-1}(a)}$

One of the many advantages of bringing the two worlds together under the umbrella of a wider world is that we can easily write the equation of any line if we know its angle with the horizontal. For instance, we can write the equation of the bisector between two lines or we can write the equation of a parallel or perpendicular passing through a given point, etc.

Given **y = 0.5x** and **y = 2x** the angles of the two lines with the horizontal are **$\tan^{-1}(0.5)$** and **$\tan^{-1}(2)$** respectively. The angle of the bisector is their average, so the equation of the bisector is:

$$y = \left(\tan \frac{\tan^{-1} 0.5 + \tan^{-1} 2}{2} \right) x$$

Based on such an approach, students perceive the construction of mathematics as a logical and easy procedure which they can control and which gives them a sense of empowerment.

Mathematical Meta-constructivism

Sometimes, when we look for a meta-constructivist path, we arrive at a crossroads of worlds as in Fig. 8. There are situations in life when we have to decide between two apparently incompatible paths. The evolutionary trajectory offers many examples of such decisions.[9]

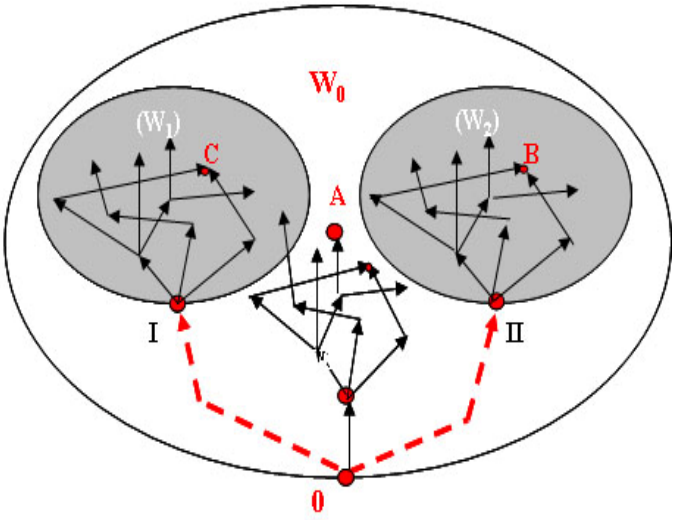

Fig. 8

An interesting example is given by the "crossroads" between real and the imaginary numbers. These numbers are defined as follows:

(1) $(-a) \times (-a) = a^2$ (real numbers)
(2) $(-a) \times (-a) = -a^2$ (imaginary numbers)

In the world of multiplication (for the real numbers - as defined in school), the imaginary numbers are incompatible. However, if we define both numbers in the world of addition, in which the world of multiplication is included, they appear as two possible distinct directions:

(1) – for real numbers

$$\underbrace{-a + -a + \ldots + -a}_{\text{-a times}} = a^2$$

(2) – for imaginary numbers

$$\underbrace{-a + -a + \ldots + -a}_{\text{-a times}} = -a^2$$

Seen from the wider world of addition, both real and imaginary numbers are possible distinct directions within the same world of complex numbers. In this new complex world, we can define (i) the unit of imaginary numbers as we defined (1) as the unit of real numbers. We can write:

A) (-1) x (-1) = (+1) and (+1) x (+1) = (+1) for real numbers – (W_1) in Fig. 9;
B) (-i) x (-i) = (-1) and (+i) x (+i) = (-1) for imaginary numbers – (W_2) in Fig. 9.

[9] Some fish decided to give up their marine habitat and fly. They switched from one habitat to an apparently incompatible one, as one cannot be both fish and bird at the same time. An example of a reverse decision is the penguin who gave up flying on earth and is preparing to return to water now.

We can now redefine the square root for both numbers, and we can combine them in the new world of complex numbers – (W_0) in Fig. 9:

$$\sqrt{1} = \pm 1 \text{ and } \sqrt{4} = \pm 2$$
$$\sqrt{-1} = \pm i \text{ and } \sqrt{-4} = \sqrt{-1} \times \sqrt{4} = \pm 2i$$

With adults, meta-constructivism, as a macro strategy, comes prior to constructivism. It is interesting to note that it takes a few minutes to make a meta-constructivist decision, but it may take months until a constructivist decision[10] can be reached.

The Strategic-constructivism

All teaching has a unique goal: enable the student to solve practical problems. At this level, the student has acquired the constructivist as well as the behaviorist tools and is supposed to use them in solving problems. The student has to find first the macro constructivist path (the logic of solving the problem) and then he has to apply the constructivist/behaviorist tools for connecting events in every world of the macro constructivist path.

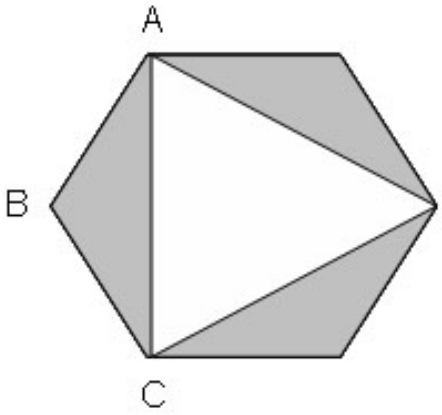

Fig 9

Let's consider the following problem. Find the area of the grey zone of a regular hexagon if the side of the inner triangle is 8 cm (see Fig. 9).

We start by deciding the *meta-constructivist path*: in which geometrical world could we place the problem. The answer is: the 'world' of triangles (see Fig. 10). The next step would be to follow the macro constructivist path while applying appropriate constructivist and behaviourist tools to every world. Finally, strategic-constructivism deals with finding the most appropriate combination of tools as a result of the dialogue between meta-constructivism and constructivism.

In the world of geometry there are standard areas (e.g. triangles) which have their own specific formulas. Since the area to be calculated is not a standard one, and therefore does not have a corresponding formula, we must calculate it as a combination of standard areas. Thus this area could be expressed as a sum of 3 triangles. We can also prove that all triangles are equal, so they have equal areas.

From here, many paths open up as the area of a triangle could be arrived at in more than one way (the geometric formula, the Heron formula or the trigonometric formula). In this particular case, the geometric formula would be a more efficient and elegant way of solving the problem. However, we need the world of trigonometry to find the height of the triangle. Thus, the sequence of the "Russian dolls" may be described as follows: general geometry (polygons), trigonometry (that part of trigonometry which intersects with geometry), geometry of the triangle, and arithmetic.

[10] In his experiments Von Aufschnaiter (1999) showed that for understanding physics it takes seconds to minutes for meta-constructivist decisions to be made compared with constructivist decisions which may take hours to months.

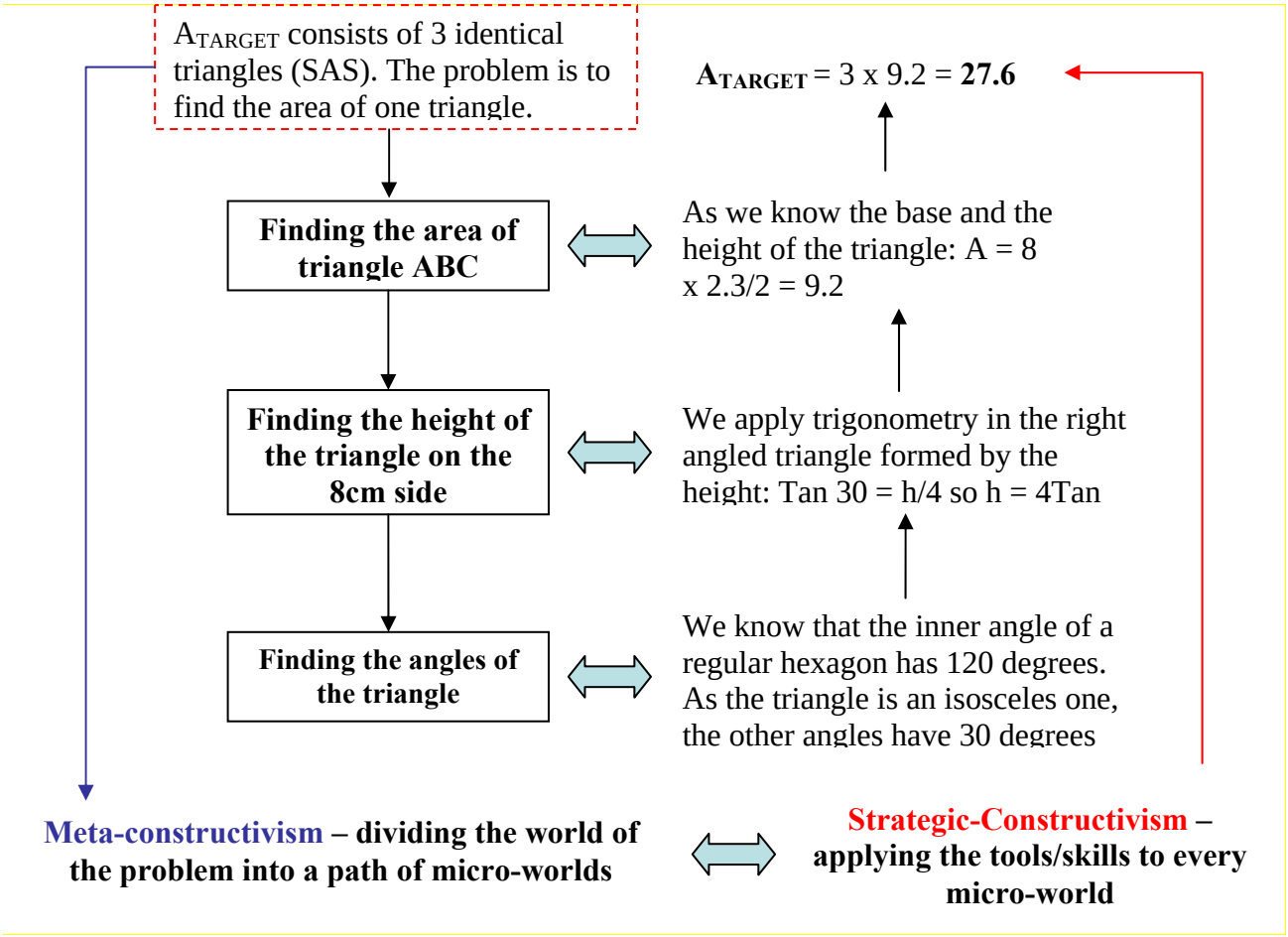

Fig. 10

Conclusions

Constructivism is usually described at different levels: at the *learning* level, where students build understanding prior to knowledge acquisition; at the *autonomy* level, where students make their own decisions; at the *conceptual* level, where students have to broaden their view in order to acquire new individual skills; at the *contextual* level, where knowledge is built through interaction; on the *experimental* level, where knowledge is given through the generative process, etc.

In the light of the above model and its applications, we suggest that the generative constructivist teaching be viewed in three consecutive stages.

1. **Early-Constructivism**, or 'the first constructivist lesson' should be acquired at home at an early age when children learn how to see the real world through their own world and not through borrowed, alien ones. At this stage, a good analytical tool would be fairy-tales. The fact that the *Lord of the Rings* is so successful is a proof that modern man is badly in need of such an exercise. Within this stage, students should learn how to build interlocutors' worlds and how to negotiate worlds where their goals could be achieved. Motivation for interaction, which makes possible any further construction, is being built now. Success could be obtained by helping the student acquire skills in the following directions:

- **build up reciprocal pragmatic models (PMs)** in order to get strategic advantages in world negotiations;
- **choose/propose a world** in which interlocutors may identify a common goal;
- **negotiate the proposed world** (except in cases when worlds are imposed);
- **negotiate an achievable goal** in an acceptable world.

We may anytime use behaviorism as a subordinate part of constructivism, due to its efficiency in building the *non-negotiable* part of the world.

2. **Meta-Constructivism** is the stage at which students learn how to particularize within a world and how to generalize a meta-world from two or more given worlds. Mathematics has an important role to play in this respect. It teaches students to be consistent within the rules of given or imaginary new world due to its strict requirement of justifying any statement within well defined *truth-conditions*. Meta-Constructivism actually represents the building up of a logical network of worlds:
   - **Choose the network of worlds** in order to integrate the events we want to analyze in known possible worlds;
   - **Choose the logical path** within the network of worlds in order to optimally connect the event with a possible goal.

3. **During Strategic-Constructivism** students learn how to combine networks of already experienced worlds. As far as problem solving is concerned, the role of the teacher is to coach the student who has full responsibility in applying his/her own constructivist and behaviorist tools to solving problems. We view the role of the teacher as follows:
   - **teach** the students how to build their own constructivist and behaviorist tools according to students' pragmatic models and the teaching goals;
   - **coach** the students to apply their own tools to problem solving.

In terms of the teacher's strategy, especially for the students who have not been exposed to Early-constructivist teaching, we suggest the following stages for training students how to use the *modeling*, *communicative* and *strategic* constructivist tools they need to acquire:

1. assist students in discovering their own world, which they can negotiate with others in search of a common ground;
2. assist students in building the world of the subject being taught as a distinct rule-generated world;
3. assist students in placing the subject being taught in a wider social context and making their own contribution to it;
4. empower students to self evaluate the extent to which they have accomplished all the above;
5. assist students in *building new, imaginary, possible worlds*. This can be done by encouraging students to make 'unusual links' in order to bring to the foreground, develop, and retain these new worlds;
6. assist students to apply the above tools to specific situations by finding logical contexts (worlds) and logical links within the context.

Although the generative constructivist principles may be applicable to the teaching of all topics, mathematics, whose subject matter is inherently generative, is perhaps the ideal topic in which the advantages of constructivism are most visible.

The question as to what the best method or what the best combination of approaches is remains to be answered by every teacher, depending on context and goals. However, the market of the 3rd millennium has already made it clear what it needs: "Employers say they are looking for 'smart' employees who can adapt to any new circumstances, promptly assessing what they need to know and learning it" (Kuhn, D, 2005, p. 12).

We believe that generative constructivism is a solution for education to assist students in identifying their own niche in a job market where "looking like everyone else" is no longer sufficient.

Reference

Adey, P. (1999). Revisiting cognitive conflict, construction, and metaconstruction, and discovering metaconstructivism. In M. Komorek, H. Behrendt, H. Dahncke, R. Duit, W. Gräber, & A. Kross, (Eds.), Research in science education: Past, present, and future. *Proceedings of the Second International Conference of the European Science Education Research Association* (E.S.E.R.A.), Vol. 1. (pp. 58-60). Kiel, Germany: IPN.

Aufschnaiter, von, C. (1999). Bedeutungsentwicklungen, Interaktionen und situatives Erleben beim Bearbeiten physikalischer Aufgaben (The development of meaning, interaction, and situated cognition in solving physics assignments). In H. Niedderer, & H. Fischler, (Eds.), *Studien zum Physiklernen* (Studies in physics learning), Vol. 3, Berlin: Logos.

Anderson, J. R. (1990). *The adaptive character of thought*. Hillsdale, NJ: L. Erlbaum Associates.

Bedrova, E. & Leong, D. J. (1996). *Tools of the mind: The Vygotskian approach to early childhood education*. Englewood Cliffs, NJ: Prentice Hall.

Bennet, J. (1976). *Linguistic behaviour*. Cambridge, England: Cambridge University Press.

Brown, P. & Levinson, S. (1987). *Politeness: Some universals in language usage*. Cambridge, England: Cambridge University Press.

Candy, P. C. (1993). Self-direction for lifelong learning. *Studies in Art Education, 34*(3) 186-188.

Dascal, M. (2003). *Interpretation and understanding*. Amsterdam: John Benjamin Publishing Company.

Deacon, T. W. (1997). *The symbolic species. The co-evolution of language and the brain*. New York: W. W. Norton & Company Inc.

Dennet, D. (1996). *Kinds of minds: Towards an understanding of consciousness*. London: Weidenfield & Nicolson.

Doer, H. & Lesh, R. (2003). A modeling perspective on teacher development. In R. Lesh, & H. Doer, (Eds.), *Beyond constructivism:Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 125-141). Mahwah, N.J.: LEA Publisher.

Fromm, E. (1987). *Psychoanalysis & ZEN Buddhism*. London: Unwin Paperbacks.

Gauker, C. (2003). *Words without meaning*. Cambridge, Massachusetts: MIT Press.

Gärdenfors, P. (2003). *How homo became sapiens: On the evolution of thinking*. New York: Oxford University Press.

Gorayska, B. & Mey, J. L. (1996). *Cognitive technology: In search of a humane interface***.** New York: Elsevier.

Jaszczolt, K. M. (2002). *Semantics and pragmatics: Meaning in language discourse*. Harlow, UK: Longman/Pearson Education.

Kamii, C. & Joseph, L. (2004). *Young children continue to reinvent arithmetic, 2nd grade: Implications of Piaget's theory*. New York: Teachers College Press.

Koch, C. (2004). *The quest for consciousness. A neurobiological approach*. Englewood, CO: Roberts & Company Publishers.

Kopytko, R. (2001). From Cartesian towards non-Cartesian pragmatics. *Journal of Pragmatics, 33*, 783-804.

Kuhn, D. (2005). *Education for thinking*. Cambridge, MA: Harvard University Press.

Larochelle, M. & Bednarz, N. (1998). Beyond epistemological correctness. In M. Larochelle, N. Bednarz, & J. Garrison, (Eds.) *Constructivism and education*. Cambridge, England: Cambridge University Press.

Laing, R. D. (1990). *The divided self*: *An existential study in sanity and madness.* London: Penguin Books.

Lecca, D. (1985). *Human communication: A model for conversational analysis. The language of Harold's Pinter's plays*. Bucharest: Bucharest University Press.

Lecca, D. (1996). A model of human communication: A pragmatic approach. In Hale, M. & Vallejo C. (Eds.), *Working papers* (pp.105-122). Montreal: Concordia University, Department of Modern Languages and Linguistics.

Lecca, D. & Lecca, O. (2005). From the trilobites towards prolegomena for a systemic model of human communication. *Issues of cognitive linguistics, 2,* 14-27.

Lomas, P. (1987). *The limits of interpretation*. *What's wrong with psychoanalysis?* Harmondsworth, England: Penguin Books.

Margineanu, N. (1975). *Psihologie logica si matematica* (Logical and mathematical psychology). Cluj-Napoca, Romania: Dacia.

Noica, C. (1986). *Scrisori despre logica lui Hermes* (Letters on Hermes' logics). Bucharest: Cartea Romaneasca.

Rescher, N. (1974). *A theory of possibility*. Oxford, England: Basil Blackwell.

Riedl, R. (1978). *Order in living organisms: A systems analysis of evolution*. New York: Wiley.

Taylor, G. R. (1984). *The great evolutionary mystery*. London: ABACUS.

Twomey, C. (2005). *Constructivism, perspectives, and practice*. New York: Teachers College Press.

Zlatev, J. (2002). Meaning = life + culture: An outline of a unified biocultural theory of meaning. *Evolution of communication, 4*(2), (253-296).